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Comparison of collisionless macroscopic models and application to the ion–electron instability

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In a first part, different macroscopic models of linear Landau damping are compared using a concise one-dimensional (1-D) collisionless formulation. The three-moment model of Chang and Callen (CC) [Phys. Fluids B **4**, 1167 (1992)] with two closure relations (complex in the Fourier space) for the viscous stress and the heat conduction is found to be equivalent to the two-moment model of Stubbe–Sukhorukov (SS) [Phys. Plasmas **6**, 2976 (1999)], which uses a single (complex) closure relation for the pressure. The comparison of the respective closure relations favors clearly the SS pressure law, which associates an anomalous resistivity to the Landau damping. In a second part, a macroscopic interpretation, with the SS model, of the ion–electron instability shows its resistive character for low and intermediate drift velocities, and the transition to the reactive Buneman limit. The pressure law for the electrons is found to verify a simple law, whereas approximate laws are discussed for the ion pressure. These laws are used to close a macroscopic model for stability analyses of nonhomogeneous plasma structures, where SS and CC models are not applicable easily.
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I. INTRODUCTION

Numerical computations of the growth rate of the ion–electron (i–e) instability in an homogeneous plasma are well known from the work of Stringer[1] and previous authors. However, theoretical analyses are limited to two asymptotic limits, the low and high drift-velocity cases, and only the second one, which leads to the Buneman instability,[2] has been treated with a macroscopic model (a cold-fluid model, more precisely). The low-drift case, which leads to the ion-acoustic instability is always treated with the Vlasov equation in textbooks.[3–5] The physical mechanisms of the instability are clearly different for the low- and high-drift limits. The Landau resonance (i.e., Landau damping) is the cause of the ion-acoustic instability, whereas its effects are negligible in the Buneman instability. Melrose explains this last one as a *reactive* instability caused by (macroscopic) density bunching.

The interest for a unified macroscopic model of the i–e instability (and other electrostatic instabilities) for the whole drift range is two-fold. First, a clear physical interpretation of the evolution of the i–e instability from the ion-acoustic to the Buneman limits, overcoming the traditional gap between micro- and macro-instabilities, is desirable. Second, a macroscopic model of the i–e instability could be of application to problems where a kinetic formulation is intractable analytically, and this includes most research problems in plasma physics. As just a single example, our actual motivation for this work came from a recent paper[6] on the stability of the plume of an electron-collecting plasma contactor, which consists of two counter-streaming plasmas separated by a strong double layer (in fact, a free discontinuity in the quasineutral plasma). The presence of two highly-accelerated beams made electrostatic instabilities very plausible, whereas the strong spatial inhomogeneity and the moving double layer made a macroscopic model almost mandatory. Using a nondissipative macroscopic model, we were able to show that the whole plasma structure did not present an ion–electron *macro*-instability, but, since Landau resonance was not included in that model, the presence of an ion-acoustic *micro*-instability could not be investigated. Recently, Chang and Callen (CC)[7] and Stubbe and Sukhorukov (SS)[8] have succeeded in deriving, from the plasma kinetic equation, macroscopic models that correctly include the effects of Landau resonance. These models (named Landau fluid models by some authors) are restricted to the linear perturbation limit, but this is enough for a macroscopic study of the i–e instability.

The first part of this paper (Sec. II) consists of a critical comparison of the CC and SS models. There are several reasons to justify it. The main one is that the two models seem different at first sight and nobody has shown that they are equivalent. Chang–Callen present a 3-moment ''viscous'' model consisting of particle, momentum and energy equations plus two closure laws for the viscous stress and the heat conduction. The more recent work of Stubbe–Sukhorukov proposes a 2-moment model, with just the particle and momentum equations and a single closure law for the pressure. These last authors claim that their model is the simplest one but a detailed comparison with the CC model is lacking. This comparison is not evident since Stubbe–Sukhorukov use a

1-D unmagnetized formulation whereas Chang–Callen depart from a 3-D, magnetized one. Besides, both formulations include collisions, which makes lengthier the model derivation and mixes collisionless and collisional effects, and use a somehow different formalism to construct the macroscopic equations from the velocity moments of the Boltzmann equation. Finally, *approximate* models (like Refs. 9 and 10) must be contrasted with the CC and SS *exact* models.

In Sec. III we use the SS model to present a unified macroscopic interpretation of the i–e instability, pursuing the goals expressed in a previous paragraph. Special attention is dedicated to the velocity range where the instability is dominated by Landau resonance. Our final goal in the paper is to discuss an approximate form of the closure relation, which can be applied reliably in stability analyses of spatially-inhomogeneous plasma structures. The SS or CC closure laws cannot be used directly, since they are derived for homogeneous plasmas and depend on the phase-velocity of the perturbations. On the other hand, the Hammett–Perkins (HP) model, for instance, is accurate only for perturbations with low phase-velocities (in each species reference frame), which, in general, is not the case for i–e modes.

II. COLLISIONLESS MACROSCOPIC MODELS

An adequate framework to highlight the effects of Landau resonance and to compare the models of Stubbe–Sukhorukov and Chang–Callen is to consider a collisionless, homogeneous plasma, and to apply perturbation theory onto the 1-D Vlasov equation for the distribution function, $f(x,v,t)$, of a certain plasma species. In the Fourier space, the response of the distribution function, $f_1(x,v,t)$ to a small electrostatic perturbation $\phi_1 \exp \mathrm{i}(kx-\omega t)$, satisfies

$$(\omega - kv) f_1 - \frac{q\phi_1 k}{m}\frac{df_0}{dv} \simeq 0, \tag{1}$$

or, clearing f_1,

$$f_1 \simeq \frac{q\phi_1 k}{m(\omega - kv)}\frac{df_0}{dv}. \tag{1'}$$

Here, subscripts 0 and 1 represent steady-state and perturbation conditions, respectively; $f = f_0(v)$ is the 1-D Maxwellian distribution function in the reference frame where the selected species is at rest; and the rest of the symbols are conventional.

Zeroth and first order, macroscopic variables are defined from 1-D, v^r-integral moments ($r=0,1,2,\dots$) of the distribution function $f(x,v,t)$. Thus, $n=\int dv\, f$ is the particle density, $nV=\int dv\, v f$ is the density flux, $p=m\int dv(v-V)^2 f$ is the (1-D) pressure, and $Q=\frac{1}{2}m\int dv(v-V)^3 f$ is the heat conduction; the temperature is $T=p/n$ and the internal energy per particle is $\mathcal{E}=T/2$, as adequate to this 1-D motion. (We will see later that Chang–Callen use different definitions for T and p, with consequences on the pressure/stress terms.)

Then, exact macroscopic equations for the perturbation problem come out from the v^r-integral moments, of either one of Eqs. (1) or (1′). Comparing the procedures followed by different authors, we find that the standard way to derive closed macroscopic models consists in the following.

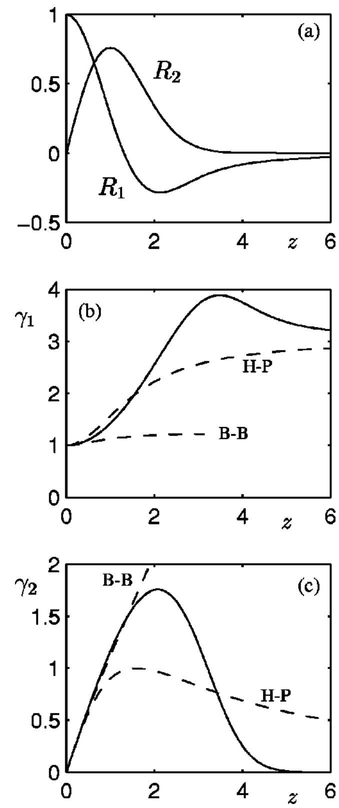

FIG. 1. (a) Real and imaginary part of the dispersion function $R(z)$ for z real. (b)–(c) Components of the polytropic complex coefficient of the SS closure law, for z real. Dashed lines correspond to the approximate models of Hammett and Perkins (HP) and of Bendib and Bendib (BB).

(a) Taking the v^0-moment of (1′), which yields the linear relation between n_1 and ϕ_1,

$$\frac{n_1}{n_0} = -\frac{q\phi_1}{T_0} R\left(\frac{\omega}{kc_0}\right), \tag{2}$$

with $c_0=\sqrt{T_0/m}$ the species thermal speed, and $R(z) = R_1(z) + \mathrm{i}R_2(z)$ the plasma dispersion function,[5]

$$R_1(z) = 1 - z\exp(-z^2/2)\int_0^z \exp(y^2/2)dy,$$

$$R_2(z) = \sqrt{\pi/2}\, z\exp(-z^2/2),$$

shown in Fig. 1(a) for z real.

(b) Taking successive v^r-moments of Eq. (1), starting from $r=0$, which yields the classical succession of conservation equations:

$$\omega n_1 = n_0 k V_1, \tag{3}$$

$$\omega m n_0 V_1 = k(q n_0 \phi_1 + p_1), \tag{4}$$

$$\omega n_0 \mathcal{E}_1 = k(p_0 V_1 + Q_1), \tag{5}$$

etc. Here, p_1 and $\mathcal{E}_1$ are related through T_1: $p_1 = n_0 T_1 + n_1 T_0$ and $\mathcal{E}_1 = T_1/2$.

Notice first that the v^0-moments of Eqs. (1) and (1′) yield independent information. Second, Eqs. (2)–(5) constitute a closed set, since, departing from Eq. (2), they determine, successively, the linear relations for $V_1(\phi_1)$, $p_1(\phi_1)$, and $Q_1(\phi_1)$. Then, Eq. (2) can be considered as the *original* closure law of the conservation equations. Nevertheless, as Stubbe–Sukhorukov point out, the linearity of Eqs. (2)–(5) allows their algebraic manipulation, existing thereby infinite, equivalent expressions of the closure law. Closure relations which depend only on plasma thermodynamic properties seem desirable, and this is readily accomplished by eliminating ϕ_1 from the linear relations $n_1(\phi_1)$, $p_1(\phi_1)$, $Q_1(\phi_1)$, etc.

A. The exact 2-moment model

Stubbe-Sukhorukov propose a model that consists just of Eqs. (3), (4) and the closure law

$$\frac{p_1}{p_0}=\gamma\frac{n_1}{n_0}, \quad \text{with} \quad \gamma(z)=z^2+\frac{1}{R(z)}, \tag{6}$$

which comes out from the linear relations $n_1(\phi_1)$ and $p_1(\phi_1)$ [Stubbe–Sukhorukov obtained Eq. (6) from the v^1-moment of Eq. (1′)]. In the SS model, the energy equation (5) is uncoupled from the other ones equations determines the heat conduction.

We comment and extend now the physical interpretation given by Stubbe–Sukhorukov of the closure law (6). Equation (6) reminds a polytropic law for the pressure but with the polytropic coefficient γ complex. For z real, the pressure coefficient γ can be decomposed in real and imaginary components:

$$\gamma_1=z^2+R_1/|R|^2, \quad \gamma_2=R_2/|R|^2, \tag{7}$$

both positive and satisfying $\gamma=\gamma_1-\mathrm{i}\gamma_2$. Figures 1(b) and 1(c) depict them as functions of ω/k. Two salient features of Eq. (6) are readily seen: first, the term γ_2 is due uniquely to Landau resonance and becomes zero only for $z\to 0$ and $z\to\infty$; second, at these two limits (of the perturbation phase-velocity) Eq. (6) recovers the isothermal ($\gamma=1$) and isentropic response ($\gamma=3$, which is the ratio of specific heats for a 1-D motion), respectively, as expected from basic physical reasoning. Other important features of the evolution of γ are (i) Landau effects on the plasma response are maximum around $z\simeq 1.212$, where γ_2/γ_1 is maximum; and $\gamma_1(z)$ does not evolve monotonically from the isothermal to the adiabatic limit, instead it presents a maximum value of $\gamma_1\simeq 3.884$ at $z\simeq 3.434$.

The main physical effect of Landau resonance is wave damping. From the density response to an electric charge perturbation, Stubbe–Sukhorukov showed that wave dissipation is related to γ_2 exclusively. A suitable way to express this result is the following form of the closure law:

$$p_1=\gamma_1 n_1 T_0-\mathrm{i}\gamma_2' m n_0 c_0 V_1, \quad \text{with} \quad \gamma_2'=\gamma_2/z, \tag{8}$$

which comes from Eqs. (3) and (6). This law, besides separating the nondissipative (or polytropic) and dissipative components of the total pressure, expresses very clearly that there is a (real) resistivity term associated to Landau resonance. Coefficients for Landau collisionality and Landau viscosity, of possible use in temporal or spatial evolution problems, would be $\nu_{\mathrm{L}}=c_0 k\gamma_2'$ and $\mu_L=c_0^2\gamma_2/\omega$, respectively.

B. The exact 3-moment model

Three-moment models are generally closed with a Fourier-like law relating heat conduction and temperature. From our relations $Q_1(\phi_1)$ and $T_1(\phi_1)$, the exact Fourier law is

$$Q_1=-\mathrm{i}\beta n_0 c_0 T_1, \tag{9}$$

with

$$\beta(z)=\mathrm{i}\frac{z}{2}\frac{(z^2-3)R(z)+1}{(z^2-1)R(z)+1}\equiv\mathrm{i}\frac{z(\gamma-3)}{2(\gamma-1)},$$

with $\beta(z)$ a complex coefficient, proportional to thermal diffusitivity. This Fourier law and Eqs. (3)–(5) constitute a closed model completely equivalent to the SS one. We do not know of any proposal of the 3-moment model expressed exactly in this form, but we show next that the 3-moment "viscous" model of Chang–Callen is equivalent to this one, once the differences in the definitions of some macroscopic variables are taken into account.

Using primes for Chang–Callen variables when they differ from ours, these authors define the species perturbation temperature as $T_1'=\frac{2}{3}\mathcal{E}_1=\frac{1}{3}T_1$ (as it is appropriate for 3-D dynamics) and the pressure as $p_1'=n_0T_1'+T_0n_1$, so that $p_1/p_0=p_1'/p_0+2T_1'/T_0$. Substituting this expression into Eq. (4), this becomes

$$\omega m n_0 V_1=k(qn_0\phi_1+p_1'+\Pi_1'), \tag{10}$$

with $\Pi_1'=2n_0T_1'$, or expressed in terms of V_1 with the aid of the conservation equations,

$$\Pi_1'=-\mathrm{i}\beta_\pi m n_0 c_0 V_1, \quad \text{with} \quad \beta_\pi\equiv\mathrm{i}2(\gamma-1)/(3z). \tag{11}$$

The Chang–Callen model (in the 1-D collisionless limit) corresponds to Eqs. (3), (5), and (10), plus closure laws (9) and (11) [Eqs. (65) and (67) in their paper]. Several observations are worth pointing out. First, Chang–Callen call $\Pi_1'=2n_0T_1'$ a "viscosity" term, but it is not a purely dissipative term, since β_π has an imaginary part (except for $z\to 0$); on the other hand, p_1' includes a dissipative contribution. As a consequence, although $p_1'+\Pi_1'$ is equal to the right-hand side of Eq. (8), there is no direct correspondence between the individual terms: polytropic and viscous terms in Eq. (8) have *real* coefficients, and represent then *real* nondissipative and dissipative contributions. In conclusion, compared to the "nonviscous" version of the 3-moment model, we find the collisionless version of the CC model misleading, since by splitting a unique (complex) effect into two parts, they are suggesting that two independent, complex transport coefficients are needed.

Returning now to the Fourier law (9), which in the end characterizes the "nonviscous" 3-moment model, three observations have to be made. Except for particular values of z: (i) the interpretation of the imaginary component of $\beta(z)$ is unclear; (ii) Landau resonance affects *both* components of the complex coefficient $\beta(z)$; and (iii) heat conduction is not due to Landau resonance exclusively. Therefore, the comparison of Fourier law with the SS pressure law (6) leads us to conclude that the SS model is simpler and provides the clearest macroscopic interpretation of the Landau resonance, as their authors claimed. In addition, point (iii) implies that heat conduction is not the macroscopic effect describing Landau resonance. The exception to observations (ii) and (iii) is the limit $z\to 0$, when $\beta(0)=\sqrt{2/\pi}$ is real and due only to the Landau term R_2.

C. Approximate models

Several authors have proposed approximate macroscopic models of the linear Landau resonance. Hammett–Perkins (HP) close Eqs. (3)–(5) with a quasi-classical, local Fourier law of the type $Q\propto \nabla T$, which, after linearizing, takes the form of Eq. (9) but with $\beta_{\rm HP}(z)\equiv\beta(0)$. Therefore, the model is exact only in the limit $z\to 0$, when it coincides with the "nonviscous" 3-moment model. However, Hammett–Perkins claim that their model does a fair job for any value of z. From the relation $\beta(\gamma)$, Eq. (4), one has $\gamma_{\rm HP}=1+2z/(z+\mathrm{i}2\beta_{\rm HP})$, as the equivalent HP polytropic coefficient. Figure 1 shows that the HP model yields errors of order one for $z=O(1)$, and of order of magnitude for $z\gg 1$: they have $\beta_{\rm HP}\neq 0$ instead of $\beta\to 0$ for the heat conduction, and, more serious, $\gamma_{2,\rm HP}\simeq 4\beta_{\rm HP}/z$ instead of $\gamma_2\simeq\sqrt{\pi/2}\,z^5\exp(-z^2/2)$, for Landau effects, Fig. 1(c). For higher accuracy, Hammett–Perkins propose to use an approximate 4-moment model. The Bendib–Bendib approximate model, also shown in Fig. 1, coincides with the expansion of the CC model for $z\ll 1$ [however, Bendib–Bendib were able to obtain Eq. (8) by eliminating the temperature in the CC equations]. Approximate models by other authors are discussed in the Chang–Callen paper.

For a linear homogeneous problem, there is no reason to use an approximate model, existing exact models of similar complexity. For nonlinear or nonhomogeneous problems, the nonexistence of a tractable inverse Fourier transform of the SS or CC closure relations is a major difficulty. Approximate expressions of the closure relations, independent of ω, or k, or both, can then be of interest. Our main point here is that a good approximation of the closure law should be problem-dependent, and take into account the characteristics of the dominant perturbation in each particular problem. For instance, for a stability analysis, the approximations of $\gamma(z)$ or $\beta(z)$ must be based on the (ω,k) characteristic of the most unstable mode. In that sense, the HP model should be used only if $z\ll 1$ for each plasma species, which is not the case, in general. In the next section we show the way to proceed for the case of the i–e instability.

III. MACROSCOPIC INTERPRETATION OF THE ION–ELECTRON INSTABILITY

The SS model is used here for a macroscopic analysis of the i–e instability. Emphasis is put in the low and medium ranges of the drift velocity, where classical analyses do not use a macroscopic approach. Applying the SS and Poisson equation to a drifting i–e plasma, the linear dispersion relation for the eigenmodes $(\omega,\mathbf{k})$, in the ion reference frame, is

$$D(k,\omega)\equiv 1+\frac{\omega_{pi}^2}{k^2c_{i0}^2\gamma_i-\omega^2}+\frac{\omega_{pe}^2}{k^2c_{e0}^2\gamma_e-(\omega-kV_0)^2}=0, \tag{12}$$

where $V_0=\mathbf{V}_0\cdot\mathbf{k}/k$ is the projection of the electron–ion drift velocity on the direction of the wavenumber vector $\mathbf{k}$; $\gamma_\alpha\equiv\gamma(z_\alpha)$ is given by Eq. (6), with

$$z_\alpha=(\omega/k-V_{\alpha 0})/c_{\alpha 0},$$

measuring the relative phase velocity; and $\omega_{p\alpha}=c_{\alpha 0}/\lambda_{\mathrm{D}\alpha}$ with $\lambda_{\mathrm{D}\alpha}$ the species Debye length. We look for the solutions $\omega=\omega(k;V_0/c_{e0},T_{i0}/T_{e0},m_e/m_i)$ of Eq. (12) with k real. Unstable modes correspond to $\omega_{im}>0$; the wavenumber and frequency of the most unstable mode will be represented by k^* and $\omega^*=\omega(k^*)$, respectively.

In the long-wavelength limit, $k\lambda_{\mathrm{D}e}\ll 1$, it is well known that the eigenmodes of Eq. (12) are the following: (i) one pair of Langmuir waves, mounted in the electron beam and satisfying $\omega-kV_0\simeq\pm\omega_{pe}(1+\gamma_e k^2\lambda_{\mathrm{D}e}^2/2)$ with $|z_e|\simeq 1/k\lambda_{\mathrm{D}e}\gg 1$; and (ii) one pair of i–e (acoustic) modes verifying

$$\omega^2/k^2\simeq\gamma_i c_{i0}^2+(\gamma_e c_{s0}^2-V_0^2 m_e/m_i), \tag{13}$$

with $c_{s0}=\sqrt{T_e/m_i}$ the plasma sound speed, based on the electron temperature. The i–e instability comes from one of these i–e modes.

For the distinguished case, T_{i0}/T_{e0}, $V_0/c_{e0}\leqslant O(1)$, Eq. (13) shows that the phase velocity of the i–e eigenmodes satisfies $\omega/k\sim c_{s0}\ll c_{e0}$. Thus, these modes move very slowly in the electron reference frame (i.e., the electron response is quasi-steady) and we can take $z_e\simeq -V_0/c_{e0}$. This is going to simplify greatly the dispersion relation of the i–e instability. First, the (complex) polytropic coefficient for electrons is independent of ω/k and depends only on the drift velocity:

$$\gamma_e\simeq\gamma_1(V_0/c_{e0})+\mathrm{i}\gamma_2(V_0/c_{e0}) \tag{14}$$

[where we used the symmetry properties of $\gamma(z)$]. Second, imposing in Eq. (12) that the electrons behave quasi-steadily the dispersion relation for the i–e modes becomes

$$\frac{\omega^2}{k^2}\simeq\gamma_i c_{i0}^2+\frac{c_{s0}^2}{k^2\lambda_{\mathrm{D}e}^2+R_1(V_0/c_{e0})-\mathrm{i}R_2(V_0/c_{e0})}, \tag{15}$$

which is the extension of Eq. (13) to any value of $k\lambda_{\mathrm{D}_e}$. More important, this dispersion relation is the generalization of the known expression of the ion-acoustic instability[3–5] to the range $V_0/c_{e0}=O(1)$, where Landau resonance is maximum. Besides, for $T_{i0}/T_{e0}\to 0$ the ion term $\gamma_i c_{i0}^2$ can be dropped, and Eq. (15) becomes explicit for $\omega(k)$ and ω_{im}^* is easy to calculate. Figure 2(a) compares $\omega_{im}^*(V_0/C_{e0})$ as given by dispersion relations (12) and (15), and shows that the quasi-steady approximation is very accurate for $V_0/c_{e0}\leqslant z_c$ with $z_c\sim 3.4$, roughly.

Stringer found numerically a change of character of the unstable modes around $V_0/c_{e0}\sim 1.3$. Equation (15) explains this change by the fact of R_1 becoming zero at $z\simeq 1.31$. For $V_0/c_{e0}<1.3$ it is $R_1>0$ and the only instability source in Eq. (15) is the Landau-based term iR_2; the instability has then a

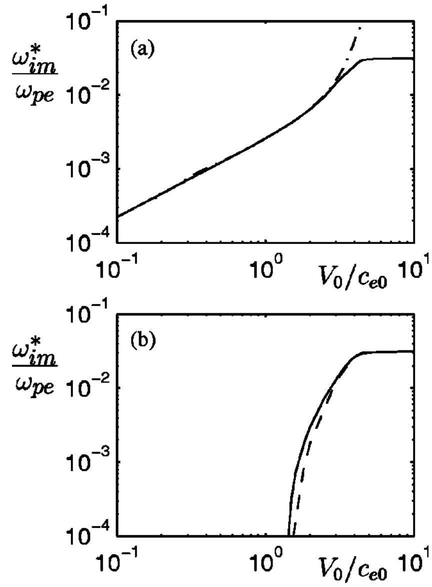

FIG. 2. Evolution of the maximum growth rate of the i–e instability with the drift velocity for (a) $T_{i0}/T_{e0}=0$ and (b) $T_{i0}/T_{e0}=1$. The dash-dot line in (a) corresponds to the approximate dispersion relation, Eq. (14). The dashed line in (b) corresponds to use the exact dispersion relation with γ_e and γ_i given by Eqs. (14) and (17).

resistive character. For $V_0/c_{e0}>1.3$, R_1 is negative and the instability presents resistive and reactive contributions. The Landau term R_2 becomes negligible for $V_0/c_{e0}>z_c$ explaining the transition to a purely reactive instability and invalidating Eq. (15). The large values of $|\omega^*|$ obtained then from Eq. (15) suggest that small inertia effects on the electrons, although small, must be retained in Eq. (12); however, it is seen that γ_e still satisfies Eq. (14) up to the Buneman limit, $V_0/c_{e0}\gg 1$. Indeed, the numerical computation of ω^*_{im} using Eq. (14) and $T_{i0}=0$ in Eq. (12) show no observable differences with the exact solution.

Physically, the presence of inertia effects means that the most unstable mode for the reactive instability is a coupled acoustic-Langmuir mode, as Stringer found out; indeed this coupling bounds the instability. On the contrary, in the resistive range the most unstable mode is not a coupled one, and the instability is bounded by the maximum of Landau resistivity.

The preceding discussion took $T_{i0}/T_{e0}=0$ and ignored the thermodynamic response of ions. For $T_{i0}/T_{e0}\ll 1$, one has $|z_i|\gg 1$, so ions behave almost adiabatically (with a exponentially small Landau damping) and the appropriate expansion of γ_i can be used to retain ion temperature effects in Eq. (15). For $T_{i0}/T_{e0}=O(1)$, it is $\Re(z_i)$, $\Im(z_i)=O(1)$, the real and imaginary components of γ_i are of the same order (but they do not correspond to γ_{1i} and γ_{2i}), and the dependence on ω/k cannot be ignored. Equation (15) is then implicit in ω/k and the most convenient form of Eq. (15) to carry out an exact computation (in the range $V_0/c_{e0}<z_c$) is

$$R(\omega/kc_{i0})=-(T_{i0}/T_{e0})[k^2\lambda^2_{\mathrm{De}}+R(-V_0/c_{e0})]. \quad (16)$$

Since an accurate expression of γ_i, dependent on the drift velocity but independent of ω/k, does not exist for the i–e modes and $T_{i0}/T_{e0}=O(1)$, we tried to find at least an approximate expression able to reproduce accurately the dominant mode of the i–e instability for $T_{i0}/T_{e0}=1$. Figure 2(b) shows that the choice

$$\gamma_i\simeq\gamma(V_0/c_{e0}) \quad (17)$$

(exact only in the instability threshold[3] at $V_0/c_{e0}\simeq 1.31$), approximates very well ω^*_{im} in the whole instability range. This leads us to propose Eqs. (14) and (17) as the best choices of γ_α in Eq. (6) for macroscopic models of the i–e instability in a one-temperature plasma.

IV. CONCLUSIONS

In the first part of the paper a concise and rigorous method to derive different exact collisionless macroscopic models has been proposed. The 2-moment model of Stubbe–Sukhorukov and the 3-moment model of Chang–Callen have been compared and found equivalent. Also, the CC model with two closure laws is found to be equivalent to the 3-moment model with just one Fourier law and no "stress" term. In the low phase-velocity limit this last model recovers the approximate model of Hammett–Perkins. The interpretation of the real and imaginary parts of the Fourier law is unclear with Landau resonance affecting both components. In addition, we find that heat conduction is not the most adequate macroscopic effect to characterize Landau resonance. On the contrary, the SS pressure law is much more amenable to physical interpretation. Extending the discussion of these authors, we have highlighted the anomalous resistivity associated to Landau resonance, and the nontrivial evolution of the plasma thermodynamic response with the perturbation phase-velocity. In conclusion, in the absence of nonlinear analyses to provide new light on the correct closure law, the SS model is the most adequate option to treat macroscopically Landau resonance, as their authors claimed.

The application of the SS model has allowed us to discuss a unified macroscopic theory of the i–e instability, overcoming the traditional gap between the *kinetic* ion-acoustic and the *fluid* Buneman instabilities. One simple dispersion relation explains the i–e instability behavior for the low and mid range of drift velocities. As the drift velocity increases from zero (and for cold ions) it is shown how the instability grows together with the Landau-based resistivity. The change of character of the instability observed by Stringer at $V_0/c_{e0}\simeq 1.3$ is explained by the appearance of a reactive contribution to the instability. The reactive character dominates completely for $V_0/c_{e0}>3.4$, roughly, when Landau resistivity becomes negligible. Inertial effects on electrons cannot be ignored from there on.

For the whole range of drift velocities the electron thermodynamic response (to the i–e modes) is accurately given by using $\gamma_e=\gamma(-V_0/c_{e0})$ in the SS closure law for the electron pressure. The ion thermodynamic response (for the most unstable i–e mode) is reasonably well-reproduced by using $\gamma_i=\gamma(V_0/c_{e0})$, for $T_{i0}/T_{e0}=1$, or $\gamma_i=3$, for $T_{i0}/T_{e0}\to 0$. These expressions constitute a good example that approximate formulations of the closure laws cannot be universal and must depend on the particular problem and plasma species.

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