On the Origin of Low Frequency Oscillations in Hall Thrusters

S. Barral* and E. Ahedo†

*Institute of Fundamental Technological Research - Polish Academy of Sciences, 00049 Warsaw, Poland
†E.T.S.I. Aeronáuticos, Universidad Politécnica, 28040 Madrid, Spain

Abstract. The breathing mode is a low frequency, longitudinal bulk instability observed in a majority of Hall thrusters. Its occurrence is accompanied by wide, regular discharge current oscillations in the $10-30$ kHz range. A concise outline of the prevailing interpretations of this mode is provided, followed by an overview of a recently proposed theory. It is eventually shown that this ionization instability is not related to the motion of the ionization front but to an ionization predator-prey cycle, the former phenomenon being rather a consequence of the latter.

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INTRODUCTION

Hall thrusters are plasma accelerators using crossed magnetic and electric fields, mainly dedicated to space applications such as satellite positioning or deep space probes propulsion. Hall thrusters have been routinely used on Russian satellites since 1972. The recent widespread adoption of this technology in the USA and in Europe has prompted an intensification of research activities related to Hall discharge instabilities.

Low frequency oscillations of the discharge current ($10-30$ kHz) are observable in most Hall thrusters for a wide range of operating conditions. Ionization mechanisms that may explain these oscillations were proposed only in the late 1990s by Baranov *et al* [1] and by Martinez-Sanchez *et al* [2]. The term "breathing" oscillations was coined soon after by Boeuf and Garrigues, who observed an apparent back-and-forth motion of the ionization front in a numerical simulation of the discharge [3]. Other theories have been proposed [4, 5], but do not appear to have gained wide acceptance. The general lack of agreement between the various interpretations has motivated a revival of theoretical investigations that ultimately led to a more rigorous and general approach based on a low frequency approximation for the dynamics of charged particles [6, 7].

CONVECTIVE WAVE VS PREDATOR-PREY CYCLE

Most interpretations of the breathing mode fall into one of the following two categories: (i) predator-prey ionization cycles and (ii) convective cycles related to the motion of the ionization front. In a predator-prey cycle, the oscillation frequency is set by the local ionization dynamics. By contrast, the main time scale involved in the convective interpretation is the time needed to replenish the ionization zone with neutrals, expected to be of the order of the transit-time for neutrals.

The prevailing predator-prey interpretation of the breathing mode was proposed by J. M. Fife and M. Martinez-Sanchez in 1997. A very simple 0D model of the ionization region was put forth, which basically consists of the two classical Lotka Volterra equations whereas:

1. electrons play the role of predators,
2. neutral particles play the role of preys,
3. the density of electrons is assumed equal to that of ions (quasi-neutrality hypothesis),
4. the velocity of escaping ions — their death rate — is constant; no ions enter the domain,
5. the velocity of incoming neutrals — their birth rate — is constant; no neutrals escape the domain,
6. the decay term for neutrals and the growth term for electrons/ions are equal and solely result from ionization.

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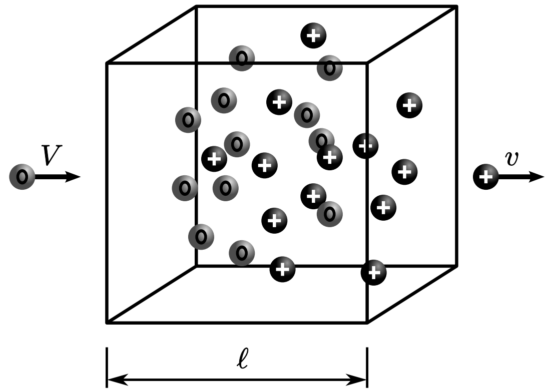

Figure 1. Conceptual representation of the ionization box of the predator-prey model. Neutral particles (0) enter the domain with velocity V. The ions (+) generated by collisions between neutrals and electrons escape the domain with velocity v. The density of electrons (not represented) is assumed equal to that of ions.

A conceptual representation of the model is given in Fig. 1. We note that hypothesis 5 implies that the flux of incoming neutrals is proportional to the density of neutrals already in the ionization region, which seems to violate causality: from general considerations about the way Hall thrusters operate, one would rather expect the flux of incoming neutrals to be constant. It turns out, however, that this hypothesis does not change fundamentally the frequency predicted by the model.

Accounting for the balance of particles inside an ionization region of length ℓ, the predator-prey model reads,

$$\ell\frac{dN}{dt} = -\ell\beta Nn + VN \tag{1}$$

$$\ell\frac{dn}{dt} = \ell\beta Nn - vn \tag{2}$$

where β is the ionization rate, N (resp. n) the density of neutrals (resp. electrons and ions) and V (resp. v) the velocity of neutrals (resp. ions). After linearization, the characteristic predator-prey frequency is obtained,

$$\omega_{pp} = \sqrt{\beta^2\bar{N}\bar{n}} = \sqrt{\frac{Vv}{\ell^2}}, \tag{3}$$

where a bar over a quantity denotes its steady state. Remembering that in most cases of interest the relation $v \gg V$ holds, it becomes obvious that the predator-prey interpretation is incompatible with the convective interpretation where the order of magnitude of the frequency is,

$$\omega_{conv} = \mathcal{O}\left(\frac{V}{\ell}\right), \tag{4}$$

that is, inversely proportional to the time needed to replenish the ionization region with neutrals.

The difference between the two interpretations, depicted on Fig. 2, can be essentially attributed to the way neutrals replenish the ionization zone. The predator-prey model implicitly assumes that neutrals replenish the whole volume uniformly; the transit time through the ionization zone plays thus no direct role. In the convective cycle, by contrast, the transit-time through the ionization zone is of key importance. Deciding upon the rightness of one or the other interpretation is a difficult task. The predator-prey interpretation is appealing by its simplicity, but the assumption that the transit-time of neutrals plays no role seems rather counter-intuitive and difficult to sustain in regards to typical simulation results. In contrast, the convective interpretation seems more apt to describe the mechanism observed in simulations, but has not received a theoretical justification yet.

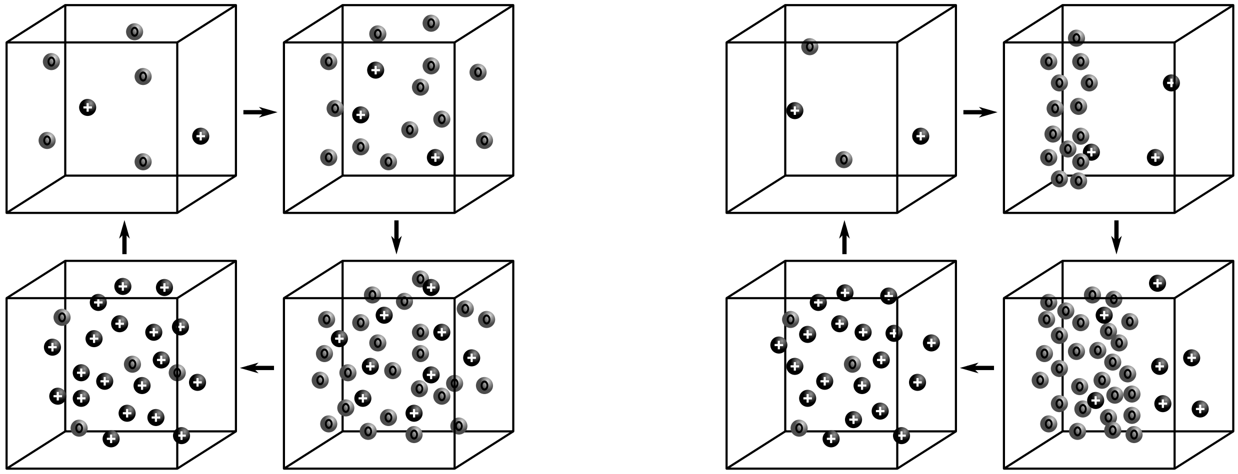

Figure 2. Predator-prey (left) and convective (right) ionization cycles.

A GENERAL THEORY

A general theory that accounts in a consistent way for the one-dimensional transport of neutrals has been developed by the authors [6, 7]. This theory is based on a low frequency approximation which in substance states that the relaxation and transit time for charged species are much smaller than that of neutrals. Assuming that the axial transport of neutrals can be described with a monokinetic conservation equation, most 1D fluid models of Hall thrusters admit a low frequency approximation that is formally expressible as [7],

$$\frac{dN}{dt}+V\frac{dN}{dx} = -I\mathscr{F}(N,\gamma), \tag{5}$$

$$N(x=0) = \frac{Q}{AVN} = \text{const}, \tag{6}$$

$$\frac{dI}{dt} = \gamma \tag{7}$$

$$U = \mathscr{U}(N,\gamma), \tag{8}$$

where $\mathscr{F}$ and $\mathscr{U}$ are respectively an operator and a functional acting on the 1D profile of N and on the instantaneous growth rate of the discharge current, defined as $\gamma = d\ln I/dt$. The cross section area of the channel A, the velocity of neutrals V and the mass flow rate at the gas feed Q are taken constant. The potential drop U is a boundary condition; it may be constant, or given as a function of the time evolution of I by a constitutive equation modeling the power supply. It must be underlined that the discharge current I in the source term of Eq. (5) actually plays the role of the density of charged species, earlier noted n: it is indeed well known from both experiments and simulations that the low frequency evolution of n and I are strongly coupled.

After linearization of the above system of equations, relatively simple relations can be derived by assuming that the the ratio of the characteristic densities of neutrals and of charged species is a small quantity,

$$\varepsilon = \mathscr{O}\left(\frac{n}{N}\right) \ll 1, \tag{9}$$

It can be noted that the former relationship actually follows from $v \gg V$ since the fluxes of neutrals and charged species are of the same order.

Assuming an ideal voltage source, $U=$ const, the leading order approximation of the oscillation frequency reads,

$$\omega \approx \sqrt{\bar{I}\frac{\partial\mathscr{U}}{\partial N}\left[\mathscr{F}(\bar{N},0)\right]\left[\frac{\partial\mathscr{U}}{\partial\nu}(\bar{N},0)\right]^{-1}}, \tag{10}$$

where derivatives are of the Fréchet type. The orders of magnitudes of the different terms can be subsequently estimated, leading to,

$$\omega = \mathscr{O}\left(\sqrt{\beta^2 \bar{N}\bar{n}}\right). \tag{11}$$

Therefore, ω is indeed of the order of magnitude of the predator-prey estimate given by Eq. (3). Although somewhat surprising at first, this fact turns out to have a simple explanation. For illustration purposes, let us assume that $\mathscr{F} = \text{const}$. After linearization of Eq. (5) and assuming harmonic oscillations around the steady state for I, the axial profile of the neutral density perturbation is thus proportional to,

$$\hat{N}(x,t) \quad \propto \quad \exp(\imath\omega t) - \exp\left(\imath\omega t - \imath\omega\frac{x}{V}\right). \tag{12}$$

This shows that the perturbation of N is the sum of a standing wave, directly related to the variations of I, and of a travelling wave, both having equal amplitudes. From that perspective, it may seem that convection (*i.e.* the travelling wave component) cannot be neglected. Intuitively, though, it can be felt that the *instantaneous* effect of $\hat{N}$ on ionization is mostly carried by its standing wave component: indeed, the travelling wave component generates at any given instant a succession of zones where $\hat{N} > 0$ and $\hat{N} < 0$, so that its average effect is to some extend cancelled. This intuitive view can be translated into a rigorous analysis based on the theory of linear functionals [6].

CONCLUSION

Low frequency oscillations result from a predator-prey cycle, whereas neutrals are preys and ions/electrons (both having the same density) are predators.

A general 1D theory shows that the variation of the density of neutrals is due to the combined effects of a standing wave and of a convective (travelling) wave. The standing wave component is related to the fact that the density of charged species —and therefore the ionization intensity— varies almost synchronously with the discharge current. The travelling wave, in turn, simply arises from the transport of perturbations by neutrals. Although both waves have similar amplitudes, the convection of neutrals plays a much smaller role in the instantaneous dynamics of the discharge because the travelling wave generates at the same time zones with an excess of neutrals and zones with a deficit of neutrals, which partially cancel one another.

In sum, the travelling waves of neutrals observed in simulations are possibly the most visible, but certainly not the most relevant aspect of breathing oscillations: neutrals created "in-place" by local ionization have a higher impact on the behavior of the discharge because their behavior is synchronized within the whole discharge. That is why the mechanism at play is better described as a predator-prey cycle than as a convective cycle.

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