Theory of a Stationary Current-Free Double Layer in a Collisionless Plasma

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Current-free double layers can develop in a collisionless, inertia-controlled plasma with two electron populations, expanding in a convergent-divergent nozzle. The double layer characteristics depend on whether they develop at the nozzle divergent side, convergent side, or throat. The divergent-geometry double layer describes faithfully the Hairapetian-Stenzel experiment [Phys. Rev. Lett. **65**, 175 (1990)], whereas the two other types correspond with those studied in self-similar expansions and wall-collection models of similar plasmas.

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A double layer (DL) consists of a positive-plus-negative Debye sheath, connecting two quasineutral regions of a small Debye-length (λ_D) plasma. Double layers have been extensively investigated in geophysics, laser-ablated plasmas, material processing, and laboratory experiments [1–3]. The strong Langmuir DL [4] is the best known of these structures. It is characterized by (i) two counterstreaming plasmas, carrying a large electric current across the DL, (ii) a DL potential fall much larger than the plasma temperatures, and (iii) a relative electric-charge density (i.e., electric-charge to ion-density ratio) of order one.

The current-free double layer (CFDL) constitutes a different subfamily. Hairapetian and Stenzel [5] reported detailed observations of a stationary CFDL in a collisionless, weakly-divergent, two-electron-population plasma. Ions enter supersonically into the DL and this contains no trapped or counterstreaming ion population, and no electron beams. The DL confines the colder electrons on the high-potential side and generates a fast monoenergetic ion beam on the low potential side, which is charge neutralized by hot electrons. The amplitude of the DL and the ion beam energy increase with the hot electron temperature, T_h. The DL moves upstream as the relative density of hot electrons increases. Contrary to the Langmuir DL, the Hairapetian-Stenzel DL is weak: the potential fall is smaller than T_h, and the relative electric charge density is small (of the order of 1%). As a consequence, the DL extends tens or hundreds of Debye lengths, which, in practice, can make it uncertain to distinguish between a ''steep quasineutral profile'' and a ''weak space-charge DL.''

Recent experiments with plasmas created by helicon sources and expanded in convergent-divergent magnetic fields have detected or inferred potential jumps downstream of the maximum magnetic field [6–9]. Although similarities with the Hairapetian-Stenzel DL are clear there are also relevant differences. First, the plasma is not collisionless and trapped low-energy ions and additional cold electrons are generated downstream of the DL. Second, the presence of hot electrons is not confirmed in these experiments (although hot electrons have been observed in other helicon-based plasmas [10,11]). Third, a steep quasineutral profile can be formed just by a highly-divergent magnetic field.

Models of a CFDL were considered in the self-similar expansion of a collisionless plasma into vacuum [12–14] and the wall collection of electronegative plasmas for material processing [15–19]. In both applications, the plasma contains two negative species with disparate temperatures. The CFDL is formed only in a limited range of temperature and density ratios of the two negative species (either hot and cold electrons or electrons and negative ions). Also, in both applications the plasma enters sonically into the DL, instead of supersonically as in the Hairapetian-Stenzel DL. At the DL exit, the flux is supersonic in the self-similar expansion, but subsonic with short-length spatial oscillations in the wall-collection model [17–19]. Therefore, three different types of CFDL have been identified.

This work presents a model of the formation of a stationary CFDL in a collisionless 3-species plasma expanding through a convergent-divergent (i.e., a de Laval) nozzle with a given cylindrical cross section $A(z)$, that can be generated by a solid wall, a magnetic field, or thermal expansion. An asymptotic two-scale analysis is carried out, based on the length hierarchy $\lambda_D \ll L \ll \lambda_{\rm col}$, with $L \sim dz/d\ln A$ the nozzle divergence length and $\lambda_{\rm col}$ the shortest mean-free path of possible collisional processes. Macroscopic equations for cold ions (i) and Maxwellian-like cold (c) and hot (h) electrons are

$$An_iu_i = \text{const}, \quad (1)$$

$m_iu_i^2/2 + e\phi = 0$, and $n_j = n_{j0}\exp(e\phi/T_j)$, for $j = c, h$, where symbols are conventional and subscript 0 refers to far-upstream conditions. In the quasineutral scale, one has

$n_i = n_e \equiv n_c + n_h$ and the plasma acts as a single fluid with $e\phi$ the specific enthalpy and $c_s(\phi) = \sqrt{(en_e/m_i)d\phi/dn_e}$ the local sound velocity, which varies along the nozzle from about $\sqrt{T_c/m_i}$ to $\sqrt{T_h/m_i}$ [12].

Dimensionless variables are $\bar{n}_{i,e} = n_{i,e}/n_0$, $\bar{u}_i = u_i(T_c/m_i)^{-1/2}$, and $\psi = -e\phi/T_c$, with $n_0 = n_{c0} + n_{h0}$. The plasma is characterized by the ratios $\alpha = n_{h0}/n_0$ and $\tau = T_h/T_c$. The (dimensionless) flux of the quasineutral plasma $g = \bar{n}_e\bar{u}_i$ satisfies $g(\psi; \tau, \alpha) \equiv \bar{n}_e\sqrt{2\psi}$ with $\bar{n}_e = (1-\alpha)e^{-\psi} + \alpha e^{-\psi/\tau}$. Regions with $dg/d\psi < 0$ and $dg/d\psi > 0$ correspond to subsonic ($u_i < c_s$) and supersonic ($u_i > c_s$) flows, respectively.

The substitution of $g(\psi)$ in Eq. (1) yields an implicit equation for the potential profile $\psi(z)$:

$$g(\psi) = g_S A_S / A(z), \tag{2}$$

where g_S is the ion flux at the throat, to be determined, and A_S is the throat area. The right-hand side of Eq. (2) presents a maximum at the nozzle throat. For the solution to be fully quasineutral and regular across the throat, g_S must be the single maximum of $g(\psi)$. For a simple 2-species plasma (i.e., only one negative species and $\alpha = 0$), one obtains $\psi_S = 1/2$, $g_S = e^{-1/2}$, and the plasma has a regular subsonic or supersonic transition at the throat with $\bar{u}_{iS} = 1$. This sonic transition at a de Laval magnetic nozzle was verified experimentally by Andersen *et al.* [20].

For a 3-species plasma, $g(\psi)$ presents 1 or 3 local extrema depending on τ and α, Fig. 1. There is a single maximum, and therefore a fully quasineutral solution, for $\tau < \tau^* = 5 + \sqrt{24} \simeq 9.90$ and any α. This is the same limit condition obtained in the self-similar expansion [12] and the spherical wall collection [15] of a collisionless plasma, and the planar wall collection of a source-driven plasma [19]. There are 3 extrema of $g(\psi)$ [named $H1$, L, and $H2$, Fig. 1] and a multivalued quasineutral solution for $\tau > \tau^*$ and $\alpha_1(\tau) < \alpha < \alpha_2(\tau)$. The boundaries, α_1 and α_2, of this parametric region are depicted in Fig. 2 and correspond to fulfilling $dg/d\psi = 0$ and $d^2g/d\psi^2 = 0$ simultaneously. [It can be shown that this double condition satisfies the limit condition $dc_s^2/d\phi + 2e/m_i = 0$ of Ref. [12] for DL formation.]

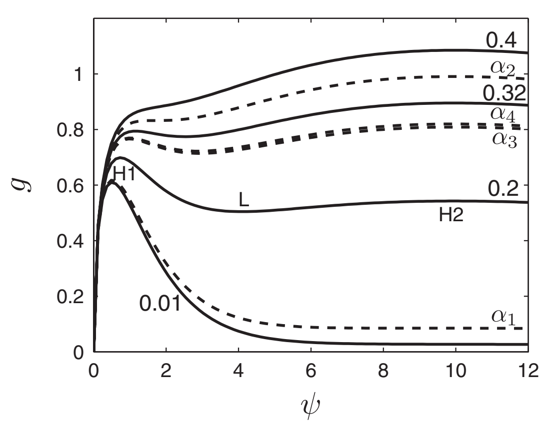

FIG. 1. Flux function $g(\psi)$ for $\tau = 20$ and different density ratios, α; distinguished values for DL formation are $\alpha_1 \simeq 0.0314$, $\alpha_3 \simeq 0.298$, $\alpha_4 \simeq 0.302$, $\alpha_2 \simeq 0.365$. Points $H1$, L, and $H2$ for $\alpha = 0.2$ illustrate the 3 extrema of $g(\psi)$.

For $\tau > \tau^*$ and $\alpha - \alpha_1(\tau) \to 0^-$, the quasineutral solution locates the nozzle throat at $H1$, and the steepened profile has $d\psi/dz \to \infty$ at $H2$, on the supersonic downstream divergent side. On the contrary, for $\alpha - \alpha_2(\tau) \to 0^+$, the nozzle throat is at $H2$, and there is a steepened profile with $d\psi/dz \to \infty$ at $H1$, on the subsonic upstream convergent side. Therefore, for $\tau > \tau^*$ and $\alpha_1(\tau) < \alpha < \alpha_2(\tau)$, we expect a DL to connect 2 single-valued quasineutral regions, with the DL location moving from the divergent to the convergent side as α increases from $\alpha_1(\tau)$ to $\alpha_2(\tau)$.

The plasma behavior inside the DL is obtained by using the inner variable $\xi = (x - x_{DL})/\lambda_D$, with $\lambda_D = \sqrt{\epsilon_0 T_c/e^2 n_0}$. Let points A and B represent the upstream and downstream ends of the DL. The dimensionless Poisson equation reads

$$d^2\psi/d\xi^2 = \bar{\rho}(\psi) = [g_A - g(\psi)](2\psi)^{-1/2}, \tag{3}$$

where $\bar{\rho} = \bar{n}_e - \bar{n}_i$ and $g_A = g(\psi_A)$ is the (constant) plasma flux along the DL. The first integral is

$$[(d\psi/d\xi)^2 - (d\psi/d\xi)_A^2]/2 = U(\psi) - U(\psi_A), \tag{4}$$

with $U(\psi; g_A, \tau, \alpha) = (2\psi)^{1/2} g_A + \bar{n}_c(\psi) + \tau\bar{n}_h(\psi)$. The formation of a DL connecting two quasineutral regions requires the fulfilment of 3 conditions *at the two DL boundaries*, A and B [1,3]: (a) plasma quasineutrality, $U'(\psi) = 0$; (b) zero space-charge electric field, $d\psi/d\xi \simeq 0$, so that $U_B = U_A$; and (c) the Bohm condition $U''(\psi) \geq 0$, which assures the development of a spatially nonoscil-

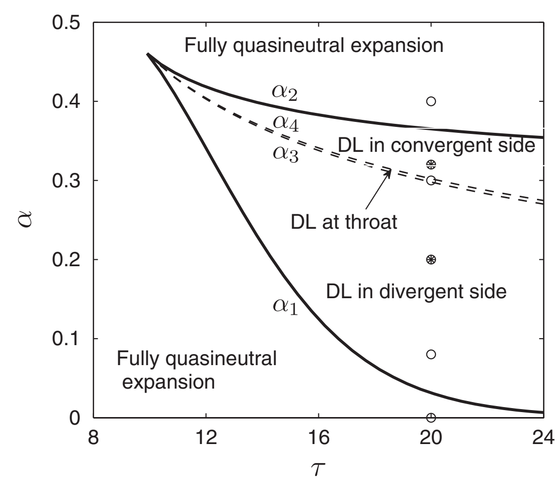

FIG. 2. Solid lines, $\alpha_1(\tau)$ and $\alpha_2(\tau)$ bound the parametric region where a DL is formed. Dashed lines, $\alpha_3(\tau)$ and $\alpha_4(\tau)$, bound regions with different DL types. Asterisks correspond to the DL profiles shown in Fig. 3. Open circles correspond to the plasma profiles shown in Fig. 4.

latory layer. In terms of the function g, these *necessary* conditions imply that $g_A = g_B$, and $g'_A, g'_B \leq 0$. Therefore, a DL connects horizontally two points A and B of $g(\psi)$ *with nonpositive slopes only*; that is, in Fig. 1, point A must be located between points $H1$ and L and point B to the right of $H2$. Finally, $U_B = U_A$, determines uniquely the DL boundaries. Physically, the ion flow at the boundaries cannot be subsonic in a standard DL. (We will see below that the violation of this condition at the DL exit leads to a more complex non-neutral structure).

A CFDL satisfying the above conditions is formed in the divergent nozzle within the parametric region $\tau > \tau^*$ and $\alpha_1(\tau) < \alpha < \alpha_3(\tau)$, plotted in Fig. 2. The condition $\alpha = \alpha_3(\tau)$ corresponds to fulfill $g_A = g_B$, $U_A = U_B$, and $dg/d\psi|_A = 0$, simultaneously. Physically, it implies that the DL entrance becomes sonic and reaches the nozzle throat, that is, point A is $H1$. Figures 3(a) and 3(b) show plasma profiles for that DL. Its characteristics coincide with those of the Hairapetian-Stenzel DL: the plasma is supersonic at the DL entrance (i.e., $u_{iA} > c_{sA}$); the DL potential jump and the downstream ion beam energy increase with T_h; the potential jump is modest in terms of T_h [$|e\phi_{AB}| \sim T_h/2$, Fig. 3(a)]; the DL confines effectively cold electrons on the upstream side, Fig. 3(b); and the relative electric-charge density is small, Fig. 3(b). From Eq. (3), the DL extension is $\sim\lambda_{Dh}\bar{\rho}^{-1/2}$, with $\lambda_{Dh} = \lambda_D\tau^{1/2}$ the Debye length based on T_h. This means an extension less than $100\lambda_D$ in most cases. For $T_c \sim 5\,\mathrm{eV}$ and $n_0 \sim 10^{18}\ \mathrm{m}^{-3}$, it is $\lambda_D \sim 2\ \mu\mathrm{m}$, so this weak DL should still be seen as a jump in the experimental data.

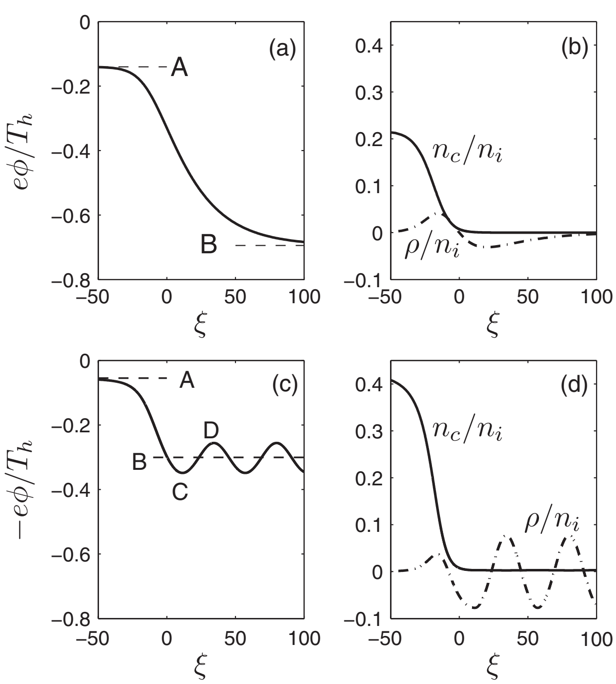

FIG. 3. Internal structure of a DL located in (a)–(b) the divergent side [$\tau = 20$, $\alpha = 0.20$], and (c)–(d) the convergent side [$\tau = 20$, $\alpha = 0.32$]. Dashed lines correspond to entrance A and exit B. These DL inner profiles are seen as discontinuities in the quasineutral profiles of Fig. 4.

In the parametric region $\alpha_3(\tau) < \alpha < \alpha_2(\tau)$ and $\tau > \tau^*$, it is impossible to fulfill all the DL formation conditions. Point A must stay at $H1$, but then the set of conditions $g_A = g_B$, $U_A = U_B$, and $g'_B \leq 0$ cannot be satisfied. A non-neutral solution started at point $H1$ leads to an "ill-ended DL" constituted by a monotonic DL followed by a space-charge rippled tail, where $\bar{\rho} = 0$ and $d\psi/d\xi = 0$ are never satisfied simultaneously. The end C of the monotonic non-neutral layer is determined from $U_C = U_A$, (i.e., zero electric field at C). Since $\bar{\rho}_C < 0$, point C cannot connect to a quasineutral solution. The rippled tail is characterized by spatial oscillations of the electric potential in the range $\psi_C < \psi < \psi_D$. The minimum potential at point C creates a barrier for electrons such that $\bar{n}_e(\psi) = \bar{n}_{eC}$ in the rippled tail. Then, in this tail the function U, Eq. (4), takes the form $U(\psi, g_A, \bar{n}_{eC}) = \sqrt{2\psi}g_A - \psi\bar{n}_{eC}$, and the zero electric field condition, $U_C = U_D$, determines ψ_D.

Figures 3(c) and 3(d) show this "DL plus rippled tail," which is the same structure found in the wall-collection models of Refs. [17–19]. Furthermore, these works carry out single-scale ($\lambda_D/L \neq 0$) integrations, based on solving the Poisson equation in the whole geometrical domain, and show that the space-charge rippling coexists with the downstream plasma expansion. Since space-charge effects are small and cancel out over several periods, the plasma can be considered as quasineutral "on the average" downstream of the monotonic part of the DL. In order to connect continuously the DL to the downstream quasineutral plasma, the end B of the DL is defined by $\bar{n}_B = \bar{n}_{eC}$ and $\bar{u}_{iB} = g_A/\bar{n}_B = \sqrt{2\psi_B}$. Then, the quasineutral density downstream of point B is $\bar{n}_e(\psi; \tau, \alpha) = (1 - \alpha) \times \exp(\psi_{CB} - \psi) + \alpha \exp[(\psi_{CB} - \psi)/\tau]$, with $\psi_{CB} = \psi_B - \psi_C$ representing the small potential barrier on electrons. This modifies the expression of $g(\psi) = \sqrt{2\psi}\bar{n}_e(\psi)$ downstream of the DL.

This convergent-geometry DL, covers completely the parametric region $\alpha_3(\tau) < \alpha < \alpha_2(\tau)$ and $\tau > \tau^*$. Indeed, two subregions are distinguishable, separated by the line $\alpha = \alpha_4(\tau)$, plotted in Fig. 2, and obtained from solving $g_A = g_B$, $U_A = U_B$, and $dg/d\psi|_B = 0$. For $\alpha_4(\tau) < \alpha < \alpha_2(\tau)$, the DL is located in the convergent nozzle, the DL exit is subsonic, and point $H2$ defines sonic conditions at the throat. For $\alpha = \alpha_4$, the DL exit becomes sonic and thus, the DL reaches the throat. For $\alpha_3 < \alpha < \alpha_4$, the DL is located at the throat, with a sonic entrance and a supersonic exit.

This completes the characterization of the different parametric regimes depicted in Fig. 2. Figure 4 shows the spatial variation of the plasma profiles and the DL location with the density ratio α, for $\tau = 20$, covering several regimes. Profiles were obtained using Eq. (2), the corresponding DL (when it exists), and (just for illustrating spatial profiles) an arbitrary nozzle shape, $A(z)$. As in the Hairapetian-Stenzel experiment, the DL location moves

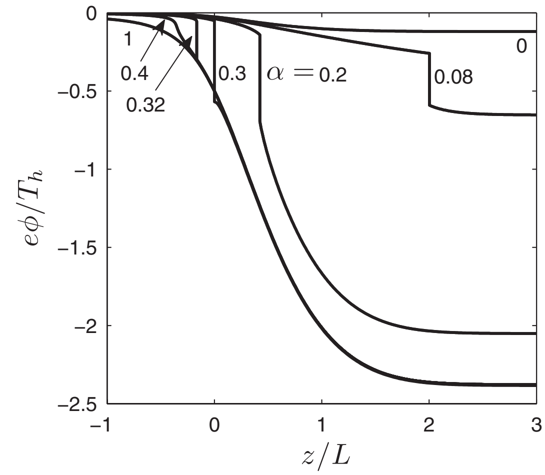

FIG. 4. Changes in plasma profiles and DL location with the density ratio for $\tau = 20$, several α, and the nozzle profile $A(z)/A_S = 3 - 2\exp(-z^2/L^2)$; the throat is at $z = 0$.

upstream as α increases. In the range $\alpha > 0.4$, profiles are identical except for a small steepening in the convergent side. The maximum potential jump across the DL is found for $\alpha = \alpha_3$. In the thin parametric range $\alpha_3 < \alpha < \alpha_4$, when the DL location crosses the nozzle throat, the quasineutral potential fall along the convergent side, i.e., ψ_S, changes by 1 order of magnitude, from $\sim 1/2$ to $\sim \tau/2$.

In conclusion, we have developed a two-negative-species plasma model with free parameters for density (α) and temperature (τ) ratios in a convergent-divergent nozzle. The exploration of the plasma behavior in (τ, α) parameter space reveals separate regimes corresponding to quasineutral expansion and DL formation. When τ exceeds about 10, three types of a stationary CFDL can be formed in a convergent/divergent geometry, depending on the density ratio α. Flux conditions at the DL entrance and exit depend on the nozzle region the layer is located: entrance and exit are supersonic in a divergent geometry; the entrance is sonic and the exit supersonic in the DL formed at the nozzle throat; and there is a sonic entrance and a subsonic exit with a space-charge rippled tail in a convergent geometry. Although the present model is specially suited for studying the acceleration of a plasma in a magnetic nozzle (a case of high interest in plasma propulsion), it provides a common frame for different cases seen before in experiments or in more restricted models. First, the divergent-geometry DL reproduces faithfully the spatial and parametric properties of the Hairapetian-Stenzel DL. Second, the convergent-geometry DL coincides exactly with the one extensively studied in wall-collection models (the nozzle throat takes here the role of the edge of the sheath adjacent to the wall). Third, the stationary sonic or supersonic DL formed at the nozzle throat is identical to the moving DL in the self-similar expansion model [12].

It is worthwhile to observe that the CFDL formation in our model is not critically related to the Maxwellian models chosen for electrons, as several works have already shown [12,16,17], but the CFDL parametric domain will depend on the selected electron model. Also, "current-free" is not a strict condition of our theory, which is valid for a "near current-free" plasma, that is one where electrons are well confined and the net electric current is much smaller than the electron thermal current.

The applicability of our CFDL theory to the Charles-Boswell experiment [6] is likely but not immediate, since their plasma is partially collisional, there are additional plasma species downstream of the DL, and wall effects can matter. In any case, our collisionless theory, based on an inertia-controlled plasma, does differ on the essentials with the diffusion-controlled theory of Lieberman-Charles-Boswell [21]. In particular, it is unclear to us the matching of an upstream diffusive (i.e., well-subsonic) plasma with their supersonic ion flow at the DL entrance.

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