Current–voltage response of anodic plasma contactors with external ionization

E. Ahedo[a)]
E.T.S.I. Aeronáuticos, Departamento Fundamentos Matemáticos, Universidad Politécnica, 28040 Madrid, Spain

(Received 28 March 1996; accepted 26 July 1996)

A spherical, steady-state model that considers the combined influence of plasma emission and external ionization of neutral gas on the operation of electron-collecting contactors is presented. Ionization by both Maxwellian and streaming electrons is investigated. A core (or quasineutral cloud) and a no-core mode are considered and the transition conditions from one to another are derived. The collected current is determined in terms of the main plasma and contactor parameters. The current-voltage response reproduces the ignited regime observed in experiments. The transition to the ignited regime and the transition to the core mode are not equivalent. It is shown that ionization by the electron beam is the main driver of ignition. The Langmuir law for the double layer causes a self-regulation of the core that affects the collection of current and leads to a multiplicity of solutions. Different neutral gas profiles are analyzed. It is uncertain whether ignition is reached when the neutral gas is provided by the contactor exclusively. The appearance of nonmonotonic potential profiles for low plasma emissions is noted. *© 1996 American Institute of Physics.* [S1070-664X(96)00911-1]

I. INTRODUCTION

Laboratory experiments and space tests[1–5] with hollow-cathode based plasma contactors suggest that external ionization of neutral gas can have a leading role in determining their current-voltage (C-V) response, in particular, in the so-called *ignited regime*. This regime is characterized by (i) a C-V characteristic with a strong enhancement of the electrical current at a nearly constant contactor voltage, and (ii) a plasma structure consisting of a glowing plasma plume (or *core*) around the contactor, separated by a space-charge sheath, the *double layer* (DL), from the background plasma (or *presheath*).

The experiments with electron-collecting contactors show that there are two electron populations in the core, a Maxwellian or confined population and an inward, high-energy beam. Both populations can ionize by electron impact the surrounding neutral gas. In most cases where ignition is reached, the potential jump across the DL is higher than the gas ionization potential, which suggests that the electron beam is the main ionization source in the ignited regime. External ionization in an electron-emitting contactor appears to behave somewhat differently[2,6] and will not be treated here.

In this paper we present a theoretical model that attempts to reproduce and discuss the ignited regime. We will proceed by including ionization in a former model[7] that disregarded the presence of neutral gas and considered that all the outward, ion current was emitted from the contactor. In this no-ionization case the plasma response depends on two dimensionless parameters, related to the contactor bias voltage and to the emitted ion current, and a *core* and a *no-core mode* of operation are possible. For a given contactor potential, a quasineutral core develops around the contactor *only when* the ion current emitted by the contactor exceeds a certain threshold, that we will call the transition to the core mode (TCM) value. For emission currents below the TCM value, the core does not form and the double layer is a sheath attached to the contactor. In all practical cases the space-charge sheath (or the DL) is very thin compared with the contactor size. Hence, on the one hand, the contactor must function in the core mode to collect large currents; on the other hand, ionization will operate differently in each mode. In the core mode the bulk of ionization is produced in the broad, quasineutral core and ionization within the DL is negligible. In the no-core mode, ionization can take place only within the sheath. A paradoxical consequence is that larger gas densities are needed for ionization to be significant in the no-core mode.

Ionization within a space-charge sheath (i.e., related to the no-core mode) was first studied by Langmuir.[8] He asserted that the rate at which ions need to be generated in order to neutralize the electron charge is about $\sqrt{m_e/m_i}$ times the rate at which the electrons are flowing (m_e and m_i are the mass of electrons and ions, respectively). From this Langmuir condition, it turns out that ionization modifies the sheath profile when the ionization mean-free path is of the order of $\sqrt{m_i/m_e}$ times the sheath thickness (provided that the potential jump across the sheath is larger than the ionization potential). The transformation of the sheath into a DL was studied numerically by Andersson and Sorensen,[9] and by Cooke and Katz.[10] Both works considered no plasma emission by the contactor and computed the relation between the contactor voltage and the gas density needed to reach the

[a)]Electronic mail: ahedo@fmetsia.upm.es

TCM. Their models did not apply to the core mode and they could not observe the transition to the ignited regime. Ionization outside a sheath was considered by Andrews and Allen[11] to derive the jump conditions across a constriction DL in a gas discharge tube.

This paper is focused on the core mode and presents a consistent determination of the C-V response in terms of the main plasma/contactor variables. The paper is organized as follows. In Sec. II we discuss the hypotheses and the equations of the model for the core mode, where ignition takes place. In Sec. III we present the C-V characteristics, analyze the plasma response, and discuss the ignition mechanism. In Sec. IV we come back to ionization in the no-core mode, present new analytical results, and compare with the core mode. In Sec. V we comment on the main results and the model restrictions.

II. FORMULATION OF THE MODEL

The contactor is a sphere of radius $r=R$ that emits plasma into another quiescent, unmagnetized plasma. In the anodic mode discussed here, the contactor is biased to a positive potential U_R relative to the undisturbed plasma. Four plasma species are considered: (e) ambient electrons, (a) ambient ions, (i) emitted ions, and (c) confined electrons. Let I_{iR} be the current of ions emitted by the contactor (which are accelerated outward by the electric field), and T_c the temperature of the electrons confined around the contactor. Contactor emissions, ionizing collisions, and other sporadic collisions would create this confined population, but its complex constitution is not studied here. We will assume, according to experimental evidence, that the c population is in Boltzmann equilibrium,

$$n_c = n_{cR} \exp \frac{eU - eU_R}{T_c}. \tag{1}$$

In general, both plasmas are partially ionized and the dynamics of the total neutral gas can be quite complex. As our purpose is to study how the neutral gas affects the C-V response, we simply assume that the steady-state density of the total neutral gas, $n_n(r)$, is known. To analyze different situations we will consider the class of potential functions,

$$n_n(r) = n_{nR}(R/r)^p, \tag{2}$$

where n_{nR} and p are given constants. For instance, $p=0$ could represent a uniform cloud of ambient gas, while $p=2$ could approximate the spherical expansion of an emitted cloud of gas.

Figure 1 illustrates the model with the three plasma regions: presheath, double layer, and core. Parametric conditions that ensure this kind of plasma structure are

$$eU_R \gg T_\infty, \tag{3}$$

$$R \gg \lambda_D (eU_R/T_\infty)^{3/4}, \tag{4}$$

$$I_i/I_e \sim \sqrt{m_e/m_i} \ll 1, \tag{5}$$

$$\frac{dU}{dr} < 0, \quad \forall r > R. \tag{6}$$

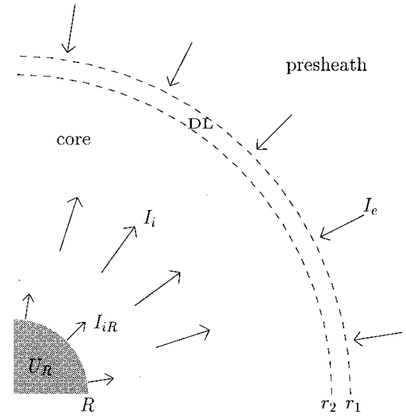

FIG. 1. Sketch of the plasma regions for the core mode; DL is double layer. Here U_R is the bias voltage; I_e is the electron current collected by the contactor; I_i is the outward ion current constituted by the ion current emitted by the contactor, I_{iR}, and the ions created by ionization within the core. When the core disappears: $r_2 \to R$, the DL becomes a sheath attached to the contactor.

Here, T_∞, n_∞, and $\lambda_D = (T_\infty/4\pi e^2 n_\infty)^{1/2}$ are the temperature, density, and Debye length of the ambient plasma, respectively, and $I_i(r)$ and $I_e(r)$ are the (counterstreaming) currents of ions and electrons, respectively ($I_\alpha = 4\pi e r^2 n_\alpha v_\alpha$, $\alpha = i, e$). The above conditions are generally satisfied in laboratory and projected space experiments.

Condition (3) assures the presence of a well-defined space-charge sheath (here, a DL) between the ambient plasma and the contactor. Outside of the sheath we have the presheath, dominated by the ambient species. From Ref. 7 we know that the transition from the presheath to the sheath occurs when $dU/dr \to \infty$, and it corresponds to the sonic point of the inward, electron population. This is the Bohm condition and yields a relationship between the electron current, I_e, and the sheath position, $r=r_1$: assuming that the e population is monoenergetic and that there is a single temperature for the ambient plasma, we have[7]

$$U_1 \equiv U(r_1) \simeq 0.75 T_\infty/e,$$
$$\frac{I_e}{4\pi R^2 e n_\infty \sqrt{2T_\infty/m_e}} \simeq j_m \left(\frac{r_1}{R}\right)^2, \quad j_m \simeq 0.43. \tag{7}$$

In the DL and core we have $eU \sim eU_R \gg T_\infty$, typically, and both the density of ambients ions and the thermal dispersion of the inwards moving electrons can be disregarded.

Condition (4) means that the contactor radius is much larger than the DL thickness, that is the double layer is quasiplanar ($r_2 \simeq r_1$ in Fig. 1) and it can be treated as a discontinuity surface between the other two quasineutral regions. Across the DL, the electric potential jumps from $U_1 \sim T_\infty/e$ to an inner value, $U_2 \equiv U(r_1^-) \gg T_\infty/e$, which must be determined. Ionization within this thin layer can be neglected.

Condition (5) states the current ratio required to sustain a stationary DL between two quasineutral plasmas. This is the Langmuir condition and has important consequences on the core structure. The first one is that changes on the electron beam due to ionization are negligible: the changes on the electron current, δI_e, are, at most, of the order of the ion current at the DL, so that, from Eq. (5), we have

$$\delta n_e/n_e \sim \delta I_e/I_e \sim I_i/I_e \ll 1.$$

Second, calling $\lambda_{\rm ion}$ to the ionization mean-free path, we estimate that

$$I_i/(r_2-R) \sim I_e/\lambda_{\rm ion},$$

so that the core length, r_2-R, is about two orders of magnitude less than $\lambda_{\rm ion}$ (the estimation is also valid if ionization is mainly produced by the confined electrons). And third, a rough estimate of the mean-free path for elastic collisions with neutrals in our range of interest is[12] $\lambda_{\rm el} \sim 10^{-1}\lambda_{\rm ion}$, yielding $(r_2-R)/\lambda_{\rm el} \sim 10\sqrt{m_e/m_i} \sim 10^{-1}$, so that elastic collisions can be neglected also. Hence, the equations for the electron beam in the DL and core are

$$I_e(r) \simeq I_e(r_1) \quad \text{(const)}, \tag{8}$$

$$4\pi e r^2 n_e \simeq \frac{I_e}{(2eU/m_e)^{1/2}}. \tag{9}$$

Condition (6) states the monotonicity of the electric potential. This is the main limitation of the model, but it is essential for a simple treatment of the ion population in the core. The distribution of the ions produced in the core is discussed in the Appendix. For steady-state ionization, the density and current of ions are obtained from Eqs. (A7), (A8), and (A10),

$$I_i = I_{iR} + \int_R^r S_i(r')\,dr', \tag{10}$$

$$4\pi e r^2 n_i = \frac{I_{iR}}{[(K_{iR}+eU_R-eU)2/m_i]^{1/2}} + \int_R^r \frac{S_i(r')\,dr'}{\{[K_i(r')+eU(r')-eU(r)]2/m_i\}^{1/2}}, \tag{11}$$

where

$$S_i(r') = n_n(\sigma_e I_e + \sigma_c 4\pi e r^2 n_c \sqrt{8T_c/\pi m_e}) \tag{12}$$

is the ion source function that takes into account ionization by both electron populations. The first term on the right of Eq. (12) is the ionization by the electron beam. Neglecting the electron thermal dispersion, the beam has an energy of eU per electron and it ionizes the gas in the region $U_I < U < U_R$, where U_I is the gas ionization potential (single-step ionization is assumed). The second ionization source on the right of Eq. (12) is the tail of the Maxwellian population of confined electrons with energy higher than eU_I. For typical conditions: $U_R/U_I \sim 1-3$, $T_\infty/eU_I \ll 1$, and $T_c/eU_I < 1$, the ionization cross sections, σ_e and σ_c, can be approximated by[12]

$$\sigma_e = \sigma_0(U/U_I - 1)H(U-U_I),$$

$$\sigma_c = \sigma_0(1+2T_c/eU_I)\exp(-eU_I/T_c),$$

with σ_0 an empirical constant (for example, in argon, $\sigma_0 \simeq 3\times 10^{-16}$ cm^2) and H the step function.

The plasma equations in the DL and core regions: $R \leqslant r \leqslant r_1$, consist of Poisson's equation,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right) = 4\pi e(n_e+n_c-n_i), \tag{13}$$

plus Eqs. (1)–(2) and (8)–(12) for the densities and currents of the four species involved. Boundary conditions for Eq. (13), and to determine r_1, are

$$r=r_1:\ U\simeq 0, \quad \lambda_D\frac{dU}{dr}\simeq 0,$$

$$r=R:\ U=U_R;$$

the asymptotic limits (3)–(4) have been considered in the first boundary condition.

Convenient variables and parameters to nondimensionalize the above equations are

$$\zeta = r/R, \quad \phi = U/U_I,$$

$$\hat{T}_c = T_c/eU_I, \quad \hat{K}_i = K_i/eU_I,$$

$$j_{e,i} = \frac{I_{e,i}}{j_m 4\pi R^2 e n_\infty \sqrt{2T_\infty/m_{e,i}}},$$

$$\nu_{cR} = (eU_I/T_\infty)^{1/2} n_{cR}/j_m n_\infty,$$

$$\nu_{nR} = n_{nR}\sigma_0 R\sqrt{m_i/m_e}.$$

Notice that ν_{nR} is both a dimensionless density and the ratio between R and the ionization typical length $\sqrt{m_e/m_i}\lambda_{\rm ion} \equiv \sqrt{m_e/m_i}(\sigma_0 n_{nR})^{-1}$.

To solve the current collection problem means to determine the collected current, j_e, the total ion current, $j_{i\infty} \equiv j_i(r \geqslant r_1)$, and the DL inner potential, ϕ_2, in terms of the contactor potential, ϕ_R, the emitted ion current, j_{iR}, the gas density profile, ν_{nR} and p, and the core temperature, $\hat{T}_c$. Besides, we must fix the values of $\hat{K}_{iR}$ and $\hat{K}_i(r')$. Parameter $\hat{K}_{iR}$ depends on the plasma ejection conditions at the (ideal) contactor surface. In the case without ionization, it was proven[7] that the ion flow had to be sonic/supersonic at that surface. Assuming that the flow was sonic, i.e., $dU/dr \to \infty$ at $r=R$, we had $\hat{K}_{iR} \simeq \hat{T}_c/2$. When ionization is significant and the solution has to be obtained numerically, that sonic condition complicates the convergence of the integration scheme; to avoid this problem we have taken $\hat{K}_{iR}$ equal to the supersonic value $\hat{T}_c$. To be consistent with the ion model of the Appendix, the ''initial'' kinetic energy, $\hat{K}_i(r')$, must be small; the constant value $\hat{K}_i(r') \equiv \hat{K}_i = 0.1$ for all r' has been used.

III. PLASMA RESPONSE AND *C-V* CHARACTERISTICS

In the core, the quasineutral limit of Eq. (13) yields for the electric potential profile, $\phi(\zeta)$, the integral equation

$$\zeta^2\nu_{cR}\exp\frac{\phi-\phi_R}{\hat{T}_c} = -\frac{j_e}{\phi^{1/2}} + \frac{j_{iR}}{(\hat{K}_{iR}+\phi_R-\phi)^{1/2}} + \int_1^\zeta \frac{s_i(\zeta')\,d\zeta'}{[\hat{K}_i+\phi(\zeta')-\phi]^{1/2}}, \tag{14}$$

where the electron and ion currents satisfy Eqs. (7)–(8) and (10),

$$j_e=\zeta_1^2=\zeta^2(\phi_2), \tag{15}$$

$$\frac{dj_i}{d\zeta}\equiv s_i(\zeta)=\frac{\nu_{nR}}{\zeta^p}\left((\phi-1)H(\phi-1)j_e + \zeta^2\nu_{cR}(1+2\hat{T}_c)\exp\frac{\phi-\phi_R-1}{\hat{T}_c}\right). \tag{16}$$

The potential at the core outer boundary, ϕ_2, is determined by just imposing that the total electric charge across the DL is zero. Integrating Eq. (13) across the quasiplanar DL and neglecting the electric field on both DL sides, we obtain

$$\left(j_e\phi^{1/2}+j_{iR}(\hat{K}_{iR}+\phi_R-\phi)^{1/2}+\int_1^{\zeta_1}[\hat{K}_i+\phi(\zeta') - \phi]^{1/2}s_i(\zeta')d\zeta'+\frac{\hat{T}_c}{2}\zeta_1^2\nu_{cR}\exp\frac{\phi-\phi_R}{\hat{T}_c}\right)_{\phi=0}^{\phi=\phi_2}=0, \tag{17}$$

where $\zeta_1^2\nu_{cR}$ is substituted from Eq. (14).

Quasineutrality at the contactor surface: $\zeta(\phi_R)=1$ in Eq. (14), yields a condition,

$$\nu_{cR}=j_{iR}/\hat{K}_{iR}^{1/2}-j_e/\phi_R^{1/2}, \tag{18}$$

among parameters at the contactor surface. Small violations of this condition are not important; it can be shown[13] that a thin space-charge sheath between the contactor and the quasineutral core adjusts parameters to satisfy Eq. (18). More important is that in order to have $\nu_{cR}>0$ in Eq. (18), the current emitted by the contactor must, at least, verify

$$j_{iR}>j_e(K_{iR}/U_R)^{1/2}. \tag{19}$$

Therefore, there is no solution for the core mode with zero plasma emission, at least for a monotonic potential. In addition, Eq. (19) is only a necessary condition and, in fact, the model fails before that condition is violated. In some cases, the failure can be due to the integration scheme, but in most cases it is due to the formation of nonmonotonic potential profiles that cannot be treated with the present model.

From Eqs. (14) and (16) the relative contribution of the two ionization sources, e and c electrons, is

$$\frac{\Delta_e j_i}{\Delta_c j_i}\sim(\phi_R-1)\exp\left(\frac{1}{\hat{T}_c}\right).$$

Figure 2 plots the parametric ranges where each ionization source dominates. Typical experimental values are $\hat{T}_c\sim 0.2-0.4$, so that e ionization is dominant when $\phi_R>1.05-1.15$.

Equation (17) is a generalized form of Langmuir law. It can be written as

$$\frac{j_e}{j_{iR}+c(j_{i\infty}-j_{iR})}=F(\phi_2,\phi_R,\hat{T}_c,\hat{K}_{iR}), \tag{20}$$

where c represents the integral term due to ionization ($c\geqslant 1$; $c=1$ for $n_n=0$) and F measures the effects of spherical acceleration; for $\hat{T}_c$, $\hat{K}_{iR}\to 0$, we have

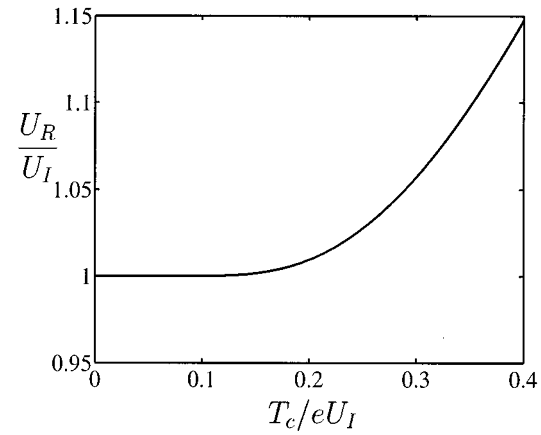

FIG. 2. Approximate regions of dominance of each ionization source: ionization by the beam electrons (e ionization) is larger than ionization by the confined electrons (c ionization) above the plotted line.

$F(\phi_2,\phi_R)\equiv\sqrt{\phi_R/\phi_2}-\sqrt{\phi_R/\phi_2-1}$. As the DL inner potential, ϕ_2, decreases, ions enter the DL with a higher kinetic energy and they are less effective in shielding the electronic charges, explaining the fact that $\partial F/\partial\phi_2>0$. A similar argument explains that ions created close to the DL are more effective, leading to $c>1$ for $n_n>0$.

Equations (14)–(18) give the plasma response and, in particular, the current-voltage characteristics,

$$j_e(j_{iR},\phi_R,\hat{T}_c,\nu_{nR},p); \tag{21}$$

note from Eq. (4) that the electron current is nearly the total current. Figure 3 plots these C-V characteristics for different plasma emissions and gas densities, when $p=0$ and $\hat{T}_c=0.1$. When there are no neutrals, $\nu_{nR}=0$, the C-V characteristic simplifies to[7] $j_e=j_e(j_{iR},eU_R/T_c)$, and the collected current presents a strong dependence on j_{iR} and a weak one on $eU_R/T_c\equiv\phi_R/\hat{T}_c$. For instance, if $eU_R\gg T_c$ and $(U_R-U_2)\ll U_R$, the C-V relation is

$$j_e\left(j_{iR},\frac{eU_R}{T_c}\right)\simeq j_{iR}\left[1-\left(\frac{T_c\ln j_{iR}}{eU_R}\right)^{1/2}\right], \tag{22}$$

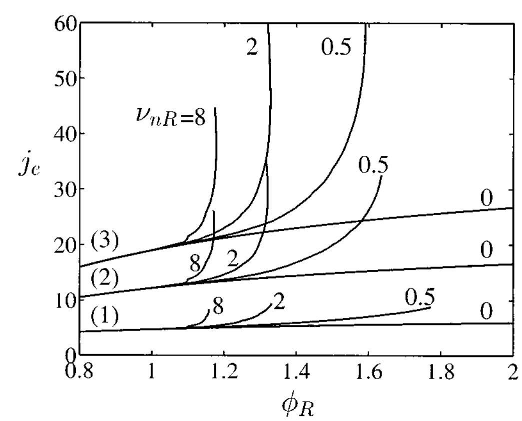

FIG. 3. C-V response for $j_{iR}=10(1)$, $30(2)$, $50(3)$ and several gas densities. Other parameters: $p=0$, $\hat{T}_c=0.1$, $\hat{K}_{iR}=0.1$, and $\hat{K}_i=0.1$.

approximately. In Fig. 3 we observe that the slope $\partial j_e/\partial\phi_R$ changes from a small to a very large value around a certain contactor voltage, $\phi_R = \phi_R^*(\nu_{nR}, j_{iR}, \ldots)$. This corresponds clearly to the transition to the ignited regime observed in experimental C-V curves. When the contactor voltage is well below the ''ignition voltage,'' ϕ_R^*, ionization appears to be weak. The plots show that ϕ_R^* depends strongly on the gas density but weakly on the emitted ion current (except when ν_{nR} is low). Another important observation is that $\partial j_e/\partial\phi_R$ can be negative, implying a multiplicity of solutions for the collection problem: two different currents, at least, can be collected with the same contactor and plasma parameters. A multiplicity of solutions was already reported when there was no ionization.[7]

We must point out that the above definition of ignition, i.e. of ϕ_R^*, is vague: there is neither a discontinuity on the function $\partial j_e/\partial\phi_R$ nor an intrinsic change on the structure of the potential profile that would permit an exact definition. On the contrary, the transition to the core mode (TCM) is well defined (for $R \gg \lambda_D$): in the core mode, it corresponds to the limit $\phi_2 = \phi_R$; from the viewpoint of the no-core mode (Sec. IV) it corresponds to the DL limit for the attached sheath. Figure 3 shows that the ignited regime and the core mode are not equivalent. We can assert only that, since j_e is close to unity throughout the no-core mode (due to the thin sheath), operation in the core mode is a necessary condition for ignition.

Figures 4(a)–4(d) give more details about the ignition process. Ignition is due to an abrupt increase of $j_{i\infty}$ [Fig. 4(a)], followed by a similar increase of j_e [Fig. 4(b)], in accordance to Langmuir law [Fig. 4(c)]. The core size increases as $j_e^{1/2}$ and this is accompanied by a smoothing of the potential profile [Fig. 4(d)] while the core potential difference, $\phi_R - \phi_2$, does not change very much. The relative contribution of each ionization source to the total ion current was also computed: since $\hat{T}_c$ is small in this case, the contribution of c ionization is negligible ($<1\%$, as long as $\phi_R > 1$). Hence, from Eq. (16) we may write $\Delta j_i \equiv j_{i\infty} - j_{iR} \propto \nu_{nR}(\phi_R - 1)$, which explains the behavior of $\phi_R^*(\nu_{nR})$ in Figs. 3 and 4(a).

Figure 5(a) shows the evolution of the collected current with ν_{nR} when ϕ_R is kept fixed, and Figs. 5(b) and 5(c) plot potential and ion-current profiles for several gas densities. We can see that the plasma adjusts its core size, depending on the gas density, in order to fulfill the Langmuir law for the currents. Thus, when the gas density is increased from $\nu_{nR} = 0$, $j_{i\infty}$ increases, and so must do $\zeta_1 \equiv j_e^{1/2}$. At the same time, as more ions are created within the core, the profile $n_i(\zeta)$, and therefore $\phi(\zeta)$, become less steep. But a smaller electric field, $d\phi/d\zeta$, means an increment in ionization, Eq. (16), that eventually leads to a reduction of the core size in order to comply with Langmuir's law. This explains the existence of characteristics with $\partial j_e/\partial\nu_{nR} < 0$ in Fig. 3, and $\partial j_e/\partial\phi_R < 0$ in Fig. 5(a).

The plasma/contactor response for a higher value of $\hat{T}_c$ is depicted in Figs. 6(a)–6(d). Currents are plotted in Figs. 6(a) and 6(b). As $d\phi/d\zeta \propto \hat{T}_c$ [Figs. 4(d) and 6(c)], larger values of $\hat{T}_c$ mean $\partial j_e/\partial\hat{T}_c < 0$ [see Eq. (22), for instance] and $\partial\phi_R^*/\partial\hat{T}_c > 0$. However, larger values of j_e/j_{iR} than in

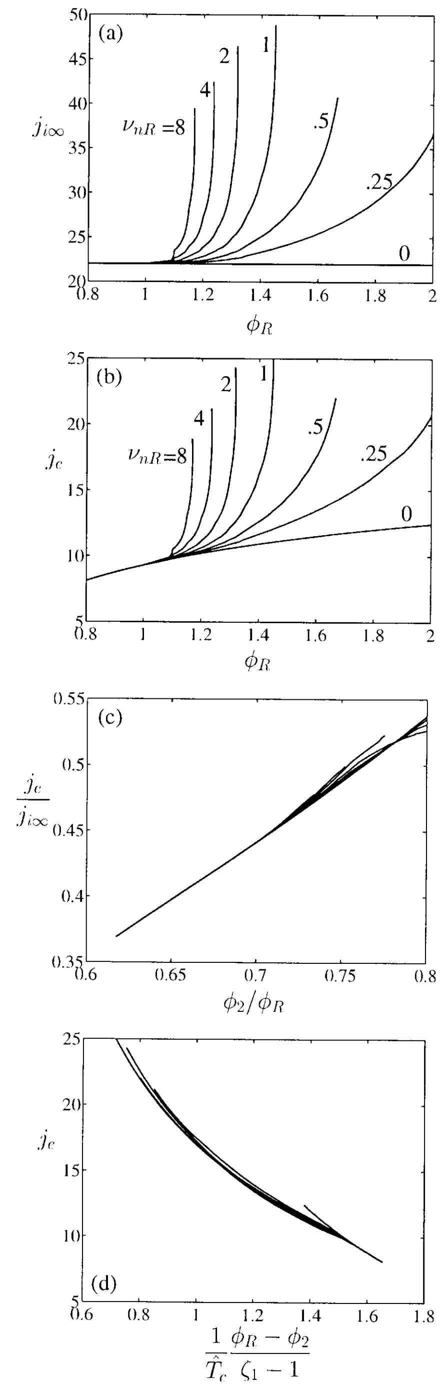

FIG. 4. Plasma/contactor response for $j_{iR} = 22$ and several gas densities; other parameters as in Fig. 3. (a) Total ion current and (b) collected current versus contactor potential; (c) Langmuir's law; (d) collected current (and core size) versus the mean slope of $\phi(\zeta)$ in the core.

Figs. 4 are attainable now, probably because of the larger electric fields. Figure 6(d) shows that c ionization is not negligible for $\hat{T}_c = 0.2$, but its contribution decreases as ignition is approached, and ignition has not been found with $\phi_R < 1$. This leads us to conclude that the monoenergetic beam is the main driver of ignition, as experimental results already suggested.

A deeper analysis of Langmuir's law gives further support to that conclusion. If we neglect c ionization in Eq. (16) and we substitute the unknown function $-\zeta^{-p}\, d\zeta/d\phi$ in the

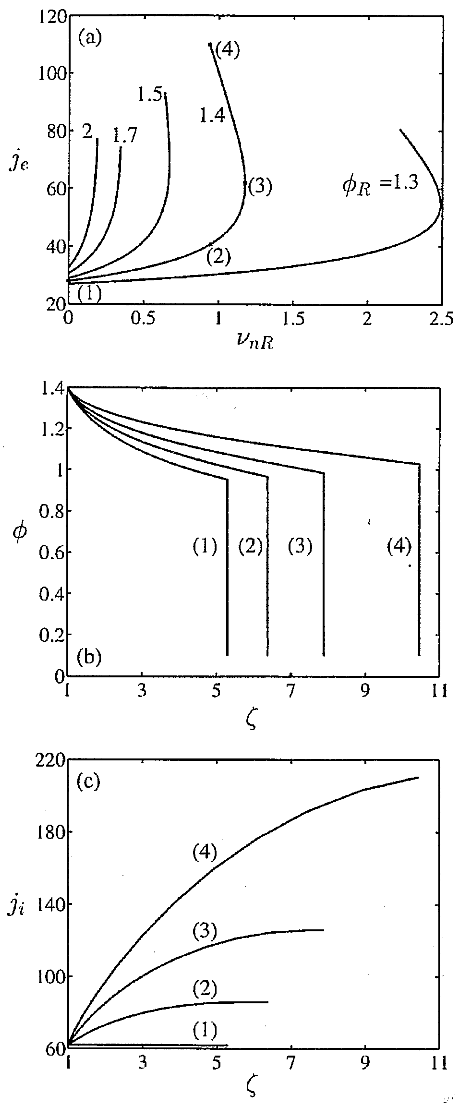

FIG. 5. (a) Collected current versus gas density for $j_{iR}=62$ and several contactor voltages. (b) Potential and (c) ion current profiles for the four operating points marked in (a). Other parameters are as in Fig. 3.

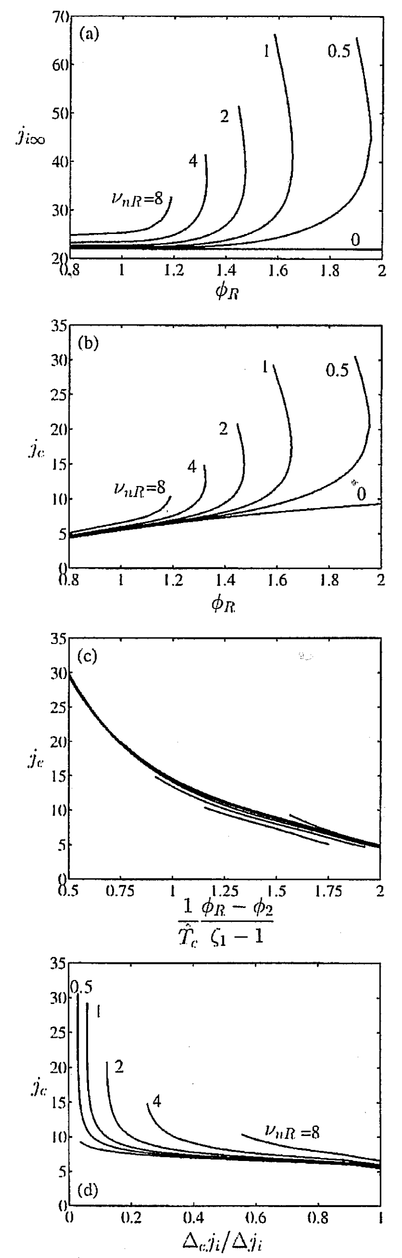

FIG. 6. Plasma/contactor response for $j_{iR}=22$ and several gas densities. Other parameters: $p=0$, $\hat{T}_c=0.2$, $\hat{K}_{iR}=0.2$, and $\hat{K}_i=0.1$. In (d) $\Delta_c j_i$ is the contribution of c ionization to $\Delta j_i \equiv j_{i\infty}-j_{iR}$.

integral term of Eq. (17) by an average value, say β, Langmuir law reads as

$$j_e=F(\phi_R,\phi_2,\hat{T}_c,\hat{K}_{iR})j_{iR}+G(\phi_R,\phi_2,\hat{T}_c,\hat{K}_i)\beta\nu_{nR}j_e, \tag{23}$$

where F was already used in Eq. (20) and G is a positive function, known in closed form. The last term in Eq. (23) is the contribution of ionization and becomes significant when $G\beta\nu_{nR}$ is of order unity. Ignition would correspond to $G\beta\nu_{nR}\simeq 1$, when Eq. (23) can be satisfied by a wide range of values of j_e, without modifying ϕ_R. The near proportionality of Δj_i with j_e in Eq. (16) is essential for that behavior. This only occurs when e ionization is dominant and explains why ignition is not found for $\phi_R<1$. On the other hand, the ignition condition $G\beta\nu_{nR}\simeq 1$ does not yield $j_e\rightarrow\infty$ because the average value β depends on $\zeta_1 \equiv j_e^{1/2}$; instead, it affects ϕ_R, yielding the negative slopes, $\partial j_e/\partial\phi_R<0$, in the C-V characteristics.

Figures 7(a) and 7(b) depict C-V responses for nonuniform gas profiles with $p=1$ and $p=2$, respectively. As p increases, ionization tends to be confined to a smaller region close the contactor, the plasma response becomes more similar to the case without ionization, and larger values of ν_{nR} are needed to generate the same ion current Δj_i [in the approximate formulation of Eq. (23) we have $\partial\beta/\partial p<0$]. The formation of a potential bump in the core inner part is more

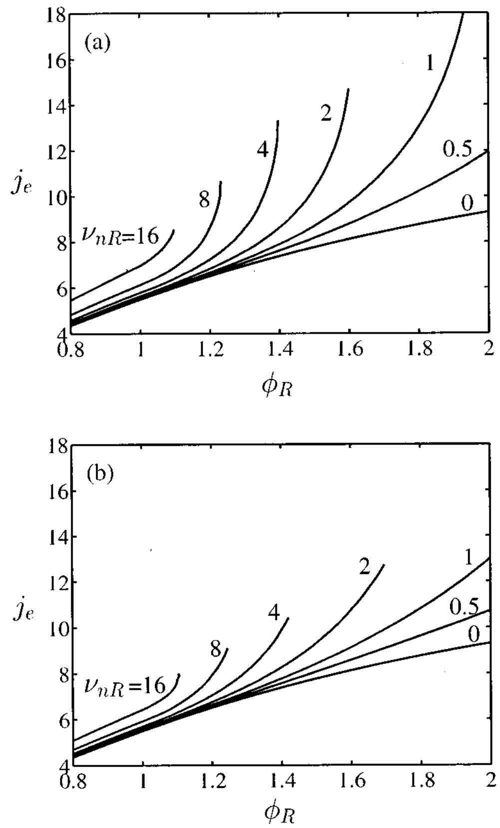

FIG. 7. C-V response for $j_{iR}=22$, several gas densities, and $p=1$ (a), 2 (b). Other parameters: $\hat{T}_c=0.2$, $\hat{K}_{iR}=0.2$, and $\hat{K}_i=0.1$.

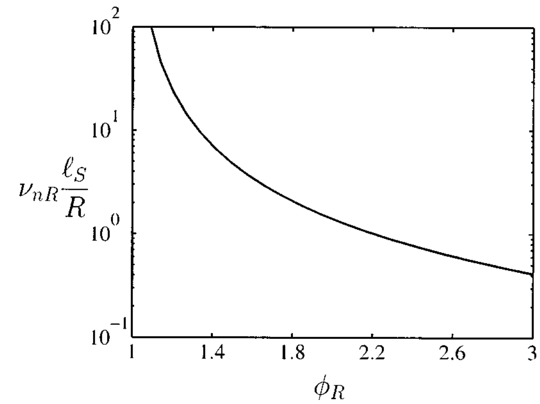

FIG. 8. Contactor potential and gas density required for the transition to the core mode (TCM) when there is no plasma emission, $j_{iR}=0$.

likely for larger p and, in fact, for $p\geqslant 2$, the calculations usually fail before ignition is reached. We cannot ascertain whether ignition occurs for $p=2$ under more general conditions; it could occur that the dependence of β on j_e in Eq. (23) would prevent the attainment of $G\beta\nu_{nR}\simeq 1$.

IV. IONIZATION IN THE NO-CORE MODE

Let us assume now that the plasma structure consists of the presheath and a space-charge sheath attached to the contactor. From the Poisson equation, Eq. (13), the typical length of the sheath is

$$l_S=\lambda_D(eU_I/T_\infty)^{3/4}j_m^{-1/2}.$$

As we have $r_1-R\sim l_S\ll R$, Eq. (3), the sheath is quasiplanar and the collected current, j_e is equal to 1 throughout the no-core mode. Therefore, the interest of this analysis lies only in the determination of the ion current and the parametric conditions for transition to the core mode (TCM). Since ionization is now produced within the thin sheath, we can take $n_n(r)\simeq\text{const}=n_{nR}$ in Eq. (6), regardless of the gas profile, p. The absence of a quasineutral core makes it very unlikely to have a substantial presence of confined electrons, and so we will take $n_c=0$ in Eq. (13).

Using $\xi=(r-R)/l_S$ as a spatial variable and

$$\nu'_{nR}=\nu_{nR}l_S/R\equiv n_n(R)\sigma_0 l_S\sqrt{m_i/m_e},$$

as the convenient nondimensional gas density, the sheath equations are

$$\mathscr{E}\frac{d\mathscr{E}}{d\phi}=\frac{1}{\phi^{1/2}}-\frac{j_{iR}}{(\phi_R-\phi)^{1/2}}-\int_1^\xi\frac{s_i(\xi')d\xi'}{[\phi(\xi')-\phi]^{1/2}},\tag{24}$$

$$\frac{d\phi}{d\xi}=-\mathscr{E}\leqslant 0,\tag{25}$$

$$\frac{dj_i}{d\xi}\equiv s_i(\xi)=\nu'_{nR}(\phi-1)H(\phi-1),\tag{26}$$

with boundary conditions

$$\phi=\phi_R:\quad \xi=0,\quad \mathscr{E}=\mathscr{E}_R\geqslant 0,\quad j_i=j_{iR},$$

$$\phi=0:\quad \xi=\xi_1,\quad \mathscr{E}=0;$$

here $\mathscr{E}_R$ and ξ_1 are unknown. Small parameters $\hat{K}_{iR}$ and $\hat{K}_i$ have been neglected from Eq. (24) because they do not introduce singularities on the solution for $\phi(\xi)$.

Using Eqs. (25)–(26), Eq. (24) can be integrated with respect to ϕ, yielding an integral equation for $\mathscr{E}=\mathscr{E}(\phi;\phi_R,j_{iR},\nu'_{nR})$,

$$\frac{\mathscr{E}^2-\mathscr{E}_R^2}{4}=\phi^{1/2}-\phi_R^{1/2}+j_{iR}(\phi_R-\phi)^{1/2}+\nu'_{nR}\int_\phi^{\phi_R}(\phi'-\phi)^{1/2}\times(\phi'-1)H(\phi'-1)\frac{d\phi'}{\mathscr{E}(\phi')},\tag{27}$$

which must be solved numerically. Then, Eqs. (25)–(26) give the potential profile, $\xi=\xi(\phi;\phi_R,j_{iR},\nu'_{nR})$, and the total ion current, $j_{i\infty}=j_{i\infty}(\phi;\phi_R,j_{iR},\nu'_{nR})$.

The transition to the core mode occurs when the electric field at the contactor surface is zero,

$$\mathscr{E}(\phi_R;\phi_R,j_{iR},\nu'_{nR})=0,\tag{28}$$

and the sheath becomes a DL. Condition (28) is both the TCM characteristic among ϕ_R, j_{iR}, and ν'_{nR}, and a limiting version of Langmuir's law. From Eq. (27) we have

$$1\geqslant j_{iR}+\nu'_{nR}\int_1^{\phi_R}\left(\frac{\phi'}{\phi_R}\right)^{1/2}(\phi'-1)\frac{d\phi'}{\mathscr{E}(\phi')},\tag{29}$$

for operation in the no-core mode, the equal sign corresponding to the TCM. As it is well known,[8] for $\nu'_{nR}=0$ the TCM occurs when $j_{iR}=1$, independently of ϕ_R (as long as the

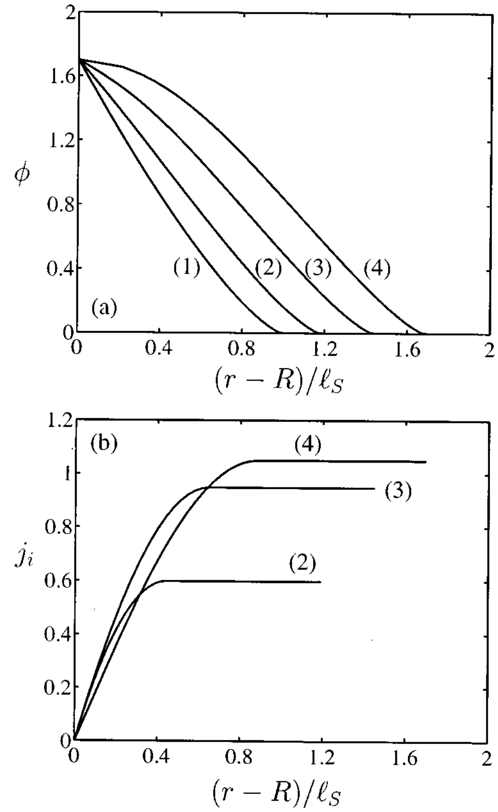

FIG. 9. No-core mode. (a) Potential and (b) ion current profiles within the sheath for $j_{iR}=0$ and $\nu'_{nR} = 0$ (1), 2.63 (2), 3.76 (3), and 3.80 (4). The last value of ν'_{nR} corresponds to the TCM.

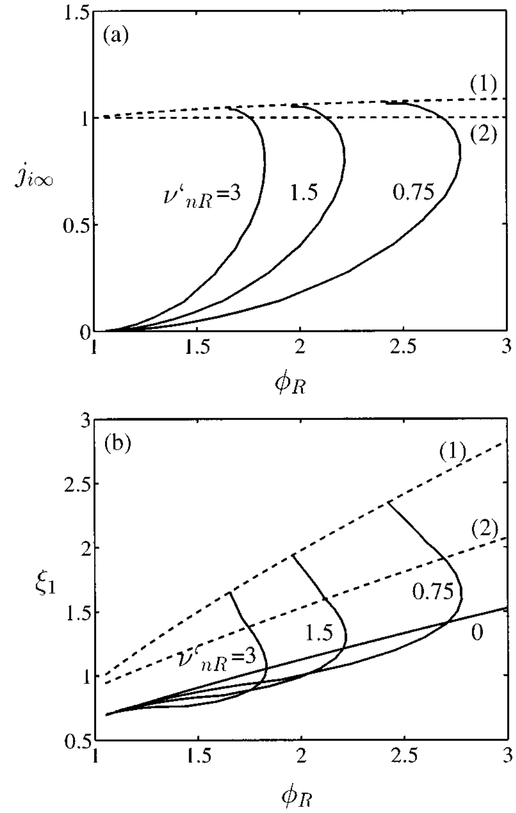

FIG. 10. No-core mode. Solid lines represent the plasma/contactor response for $j_{iR}=0$ and several gas densities. Dashed lines represent the TCM for (1) no ion emission, $j_{iR}=0$, and (2) no ionization, $\nu'_{nR} = 0$.

sheath is thin). In the case of no plasma emission, $j_{iR}=0$, the TCM characteristic, $\nu'_{nR}(\phi_R)$, is depicted in Fig. 8 (this is the case investigated in Refs. 9 and 10). For $\phi_R-1\ll 1$, its asymptotic behavior is

$$\nu'_{nR}(\phi_R)\simeq\sqrt{27\pi/32}(\phi_R-1)^{-7/4}.$$

The TCM line in Fig. 8 must be understood, in the sense that the region above this line cannot be reached by the no-core solution. For a comparison with values of the core mode, recall that $\nu'_{nR}\sim 1$ means $\nu_{nR}\sim R/l_S\gg 1$.

Figure 9 plots $\phi(\xi)$ and $j_i(\xi)$ for several gas densities and no plasma emission, $j_{iR}=0$. We observe that ionization effects are weak until ν'_{nR} approaches the TCM value. Figures 10(a) and 10(b) depict, for several gas densities and $j_{iR}=0$, the evolution of the ion current, $j_{i\infty}$, and of the sheath thickness, ξ_1, with ϕ_R, from $\phi_R<1$ (a passive contactor, actually) to the TCM. As the increase of the collected current, $\Delta j_e=j_e-1$, is approximately proportional to the sheath thickness,

$$\Delta j_e\simeq 2\xi_1 l_S/R\ll 1,$$

Fig. 10(b) can be viewed as the C-V characteristics in the no-core mode. The existence of a maximum value of ϕ_R in the C-V curves indicates that multiple solutions to the collection problem are possible within this mode also. The maximum on the voltage is justified by the evolution of the electric field, $\mathscr{E}$, at the contactor neighborhood: when $\mathscr{E}_R$ becomes small, smaller values of ϕ_R are needed to obtain the ion current required to shield the electron charge.

The case of no plasma emission, $j_{iR}=0$, could correspond to a practical device consisting of a passive sphere immersed in a partially ionized plasma (such as the TSS-1 tethered satellite). Notice that the no-core solution cannot be continued into our model of the core mode because the quasineutrality condition, Eq. (18), cannot be fulfilled. It is then unclear what the potential structure is when parameters are above the TCM line of Fig. 8, although a core with a nonmonotonic potential could be a possible structure. We do not know whether the core formation will be accompanied by ignition in that case.

V. DISCUSSION

A model that successfully reproduces C-V characteristics of contactors evolving from a nonignited regime to an ignited one has been developed. Ignition occurs within the core mode at a particular contactor voltage that depends mainly on the gas density. We have attempted to show how the electron beam is the main driver of ignition and we have shown how the plasma adjusts the core size to comply with Langmuir's law. A consequence of this self-regulation is a multiplicity of possible currents for given voltage. Therefore, the way contactor parameters can be controlled should receive more attention because it will determine the operation point. In that sense we find, for instance, that (a) in many experiments there is no direct control over the contactor bias voltage; (b) ϕ_R and ν_{nR} seem not to be independent in the ignited regime; (c) we expect a certain relation between j_{iR} and ν_{nR} when the neutral gas is provided by the contactor; and (d) actual boundary conditions for the emitted plasma are difficult to formulate. The stability of each stationary solution should be addressed also.

It is uncertain whether ignition is reached when the contactor provides most of the neutral gas ($p\sim 2$) and, in any case, the gas is spent very inefficiently. This affects applications of plasma contactors in the ionosphere, where the ambient density of neutrals is small. For such applications attention should be given to increasing the ion current emitted by the contactor.

The main restriction of the present model is the monotonicity of the potential profile. A more general model should deal with nonmonotonic structures for the core. This will permit (i) continuation of the C-V characteristics into larger values of j_e/j_{iR}, (ii) matching of the core and no-core modes, and (iii) analysis of ignition for $p=2$. The main difficulty of that model is obtaining consistent and tractable equations for the streaming ions. In any case, the plasma/contactor response in that model should not be very different from the one presented here. For instance, if the potential presents a bump in the contactor neighborhood (which is the most plausible nonmonotonicity), the present monotonic model will be approximately valid beyond that bump, and we can imagine the bump region as a prolongation of the contactor where enough ion current, j_{iR}, is generated to sustain the remaining monotonic part of the core.

ACKNOWLEDGMENTS

The author is greatly thankful to Professor M. Martínez Sánchez for his consultation and advice.

This work was supported by the Ministerio de Educación y Ciencia of Spain (Project No. PB94-0417-C03-03).

APPENDIX: DISTRIBUTION OF IONS IN THE CORE

The steady-state distribution function for the ions satisfies

$$\frac{Df_i}{Dt}\equiv \mathbf{w}\cdot\nabla_{\mathbf{r}}\, f_i-\frac{e}{m_i}\,\nabla_{\mathbf{r}}U\cdot\nabla_{\mathbf{w}}f_i=C(f_i), \tag{A1}$$

where $\mathbf{w}$ is the (individual) ion velocity and $C(f_i)$ is the collision operator that includes the source terms due to ionization. For our spherically symmetric model let δf_i be the contribution to f_i of ionization in the spherical region between r' and $r'+\delta r'$. If we neglect elastic collisions of the ions once they are created, δf_i is an arbitrary function of the two constants of the motion: the total energy E and the angular momentum J, which satisfy

$$E=\frac{m_i}{2}\,(w_r^2+w_\perp^2)+eU(r),$$

$$J=m_i r w_\perp,$$

with $w_r\equiv\mathbf{w}\cdot\mathbf{1}_r$ and $\mathbf{w}_\perp=\mathbf{w}-w_r\mathbf{1}_r$. Let us write $\delta f_i(E,J)$ as the sum of $\delta f_i^+(E,J)$ and $\delta f_i^-(E,J)$, corresponding to ions created with $w_r(r')>0$ and $w_r(r')<0$, respectively. As $dU/dr<0$, we have $Dw_r/Dt>0$ for any ion: ions with $w_r(r')>0$ move outward all the time; ions with $w_r(r')<0$ start moving inward until they reach a position $r^*(E,J)$, given by

$$E=\frac{J^2}{2m_i r^{*2}}+eU(r^*),$$

and then move outward.

Let $\delta n_i=\int \delta f_i\, d\mathbf{w}$ and $\delta(n_i\mathbf{v}_i)=\int \delta f_i \mathbf{w}\, d\mathbf{w}$ be the contributions of δf_i to the ion density and current, respectively. Using E and J as variables of integration we have[14]

$$\delta n_i(r>r')=\frac{\pi\sqrt{2}}{m_i^{5/2}r^2}\int dE\ dJ^2 \times\frac{\delta f_i^+ +\delta f_i^-}{[E-eU(r)-J^2/2m_i r^2]^{1/2}},$$
$$\delta n_i(r<r')=\frac{\pi\sqrt{2}}{m_i^{5/2}r^2}\int dE\ dJ^2 \times\frac{2\ \delta f_i^-}{[E-eU(r)-J^2/2m_i r^2]^{1/2}}, \tag{A2}$$

$$\delta(n_i\mathbf{v}_i)(r>r')=\mathbf{1}_r\,\frac{\pi}{m_i^3 r^2}\int dE\ dJ^2(\delta f_i^+ +\delta f_i^-),$$
$$\delta(n_i\mathbf{v}_i)(r<r')=0, \tag{A3}$$

where the domain of integration is $0<E<\infty$ and $0<J^2<G(E)\equiv 2m_i r^2[E-eU(r)]$. Notice that we have assumed that ions are reflected by the contactor when $r^*\leqslant R$.

In this work we are going to neglect $\delta n_i(r<r')$. For a monotonic potential, there are two cases, at least, where that simplification is justified. The first one is when most neutrals are emitted from the contactor; then, the created ions practically conserve the outward velocity of the neutrals and $\delta f_i^-\ll\delta f_i^+$. The second one is when the "initial" kinetic energy, $K_i(r')$, of the ions is small compared with the variations of $U(r)$; then, the inward region of influence, $r'-r^*$, is small,

$$\frac{r'-r^*}{r'}\sim\frac{K_i}{er'}\left|\frac{dr}{dU}\right|_{r'}\ll 1,$$

approximately. Let us assume now that, at r', all ions are created with the same kinetic energy, $K_i(r')$, and with a uniform distribution of angular momentum. Then,

$$\delta f_i(E,J)\propto\delta_{\mathrm{D}}[E-K_i(r')-eU(r')] \tag{A4}$$

(δ_{D} means the Dirac function), and introducing Eq. (A4) in Eqs. (A2)–(A3), we obtain, for $r>r'$,

$$\delta(r^2 n_i v_i)=\text{const}=\delta(r^2 n_i v_i)|_{r'}, \tag{A5}$$

$$r^2\ \delta n_i=\frac{2\,\delta(r^2 n_i v_i)|_{r'}}{\tilde{v}_i(r',r)+[\tilde{v}_i(r',r)^2-2K_i(r')r'^2/m_i r^2]^{1/2}}, \tag{A6}$$

where $m_i\tilde{v}_i^2(r',r)/2=K_i(r')+eU(r')-eU(r)$ is the kinetic energy at r of those ions created at r'.

If $C(f_i)$ in Eq. (A1) only includes ionization by electron–neutral impact, the ion continuity equation is

$$\frac{Dn_i}{Dt} \equiv \frac{1}{r^2}\frac{d}{dr}(r^2 n_i v_i) = \int C(f_i) d\mathbf{w} = n_n \nu_{\text{ion}}, \tag{A7}$$

where ν_{ion} is the ionization frequency. Then, the constant in Eq. (A5) is

$$\delta(r^2 n_i v_i)|_{r'} = (n_n \nu_{\text{ion}})(r') r'^2 \ \delta r'.$$

The total density and current of ions at a given position r is the sum of the different contributions at $R \leqslant r' < r$:

$$(r^2 n_i v_i)(r) = (r^2 n_i v_i)|_R + \int_R^r (n_n \nu_{\text{ion}})(r') r'^2 \ dr', \tag{A8}$$

$$r^2 n_i(r) = \frac{2(r^2 n_i v_i)|_R}{\tilde{v}_i(R,r) + [\tilde{v}_i(R,r)^2 - 2K_{iR}R^2/m_i r^2]^{1/2}} + \int_R^r \frac{2(n_n \nu_{\text{ion}})(r') r'^2 \ dr'}{\tilde{v}_i(r',r) + [\tilde{v}_i(r',r)^2 - 2K_i(r') r'^2/m_i r^2]^{1/2}}, \tag{A9}$$

where the first term on the right of both equations is the contribution of the ions emitted by the contactor, and $K_{iR} \equiv K_i(R)$. When $K_i(r')$ is small and $dU/dr < 0$, the effect of the ''initial'' angular momentum in Eq. (A9) is appreciable only locally at $r \simeq r'$. Neglecting this local effect, Eq. (A9) becomes

$$r^2 n_i(r) = \frac{(r^2 n_i v_i)|_R}{\tilde{v}_i(R,r)} + \int_R^r \frac{(n_n \nu_{\text{ion}})(r') r'^2 \ dr'}{\tilde{v}_i(r',r)}, \tag{A10}$$

which is the solution of Eqs. (A2)–(A3) when all ions are created without angular momentum, i.e., when

$$\delta f_i(E,J) \propto \delta_{\text{D}}(J^2)\, \delta_{\text{D}}[E - K_i(r') - eU(r')].$$

[1] J. D. Williams and P. J. Wilbur, J. Spacecr. **27**, 634 (1990).
[2] J. D. Williams and P. J. Wilbur, J. Spacecr. **29**, 820 (1992).
[3] G. Vannaroni, M. Dobrowolny, E. Melchioni, F. De Venuto, and R. Giovi, J. Appl. Phys. **71**, 4709 (1992).
[4] L. Conde and L. León, Phys. Plasmas **1**, 2441 (1994).
[5] B. E. Gilchrist, P. M. Banks, T. Neubert, P. R. Williamson, N. B. Myers, W. J. Raitt, and S. Sasaki, J. Geophys. Res. **95**, 2469 (1990).
[6] D. Parks, I. Katz, B. Buchholtz, and P. Wilbur, J. Appl. Phys. **74**, 7094 (1993).
[7] E. Ahedo, M. Martínez-Sánchez, and J. R. Sanmartín, Phys. Fluids B **4**, 3847 (1992).
[8] I. Langmuir, Phys. Rev. **33**, 954 (1929).
[9] D. Andersson and J. Sorensen, J. Phys. D Appl. Phys. **16**, 601 (1983).
[10] D. L. Cooke and I. Katz, J. Spacecr. **25**, 132 (1988).
[11] J. G. Andrews and J. E. Allen, Proc. R. Soc. London Ser. A **320**, 459 (1971).
[12] Y. P. Raizer, *Gas Discharge Physics* (Springer-Verlag, Berlin, 1991).
[13] E. Ahedo and V. Lapuerta, Phys. Plasmas **2**, 3252 (1995).
[14] I. Bernstein and I. Rabinowitz, Phys. Fluids **2**, 112 (1959).